From bhargava@math.Princeton.EDU Sat Jun 05 20:28:49 1999
Received: from tea.Princeton.EDU (bhargava@tea.Princeton.EDU [128.112.16.7])
Date: Sat, 5 Jun 1999 23:27:04 -0400 (EDT)
From: Manjul Bhargava <bhargava@math.Princeton.EDU>
To: jagy@msri.org
Subject: regular ternary forms

Dear Prof. Jagy,

I just saw your paper with Kaplansky and Schiemann on regular ternary quadratic forms--do you have proofs written now for regularity? If so, I would love to receive a preprint! Also, if you could send me (electronically) a list of the 913 regular ternary forms, that would be wonderful. (In a way that Magma can read them preferably, but any way that you have them is fine...).

Thanks so much--

Regards,
Manjul

From bhargava@math.Princeton.EDU Fri Jun 11 15:13:39 1999
Date: Fri, 11 Jun 1999 18:13:00 -0400 (EDT)
From: Manjul Bhargava <bhargava@math.Princeton.EDU>
To: William Jagy <jagy@msri.org>
cc: kap@msri.org
Subject: Re: regular ternary forms
MIME-Version: 1.0

Dear Profs. Jagy and Kaplansky,

Thanks for your note. I am actually at Princeton--my e-mail address is bhargava@math.princeton.edu . Sorry about the omission--I'm not sure how it happened...

When I hadn't heard from you for several days, I decided to run the computation myself. So I don't need an electronic list of those forms anymore. But if you have one, please do send it along anyhow.

By the way, the results of my computation seem to differ from yours a bit; I find 795 forms unique in their genus. I think you must have missed one?

Thanks again--

Best regards,
Manjul

Date: Mon, 14 Jun 1999 10:41:03 -0700 (PDT)
From: kap@msri.org (Irving Kaplansky)
To: bhargava@math.princeton.edu, kap@msri.org
Subject: Welcome to the world of ternary forms

Dear Manjul (Forgive me for presuming on a very slight acquaintance or rather none at all):

Will Jagy and I were pleased to hear from you. I believe he hasalready sent yothe list of 9 13 and his preprint of proofs. Now I will try to cover the things that occur to me.

1. We tried hard to week the work accurate but mistakes will happen.

2. Our objective was to find all the regulars; we picked up the forms alone in their genera (we have a private nickname: "loners") incidentally. However as a separate check Schiemann, at our request, ran a program to catch loners. In the odd case it all checked beautifully. In the even case the discriminants got so large that they outran his program. We therefore finished the job ourselves, including identifying the remaining even loners. If we have indeed missed a loner I predict that it is an even one with a big discriminant.

3. Naturally, it is important to identify the loner we may have missed. In addition we owe the world (or at least the tiny handful who are interested) a careful comparison of the 794. This may be tedious. Maybe you or Will can write a program to do it. I am willing to do it by hand. The following point may arise: undoubtedly some of the forms are presented in different bases. We have an excellent program here for testing equivalence of forms. It runs almost instantaneously, even for the large discriminants involved here.

4. By the way, I tried hard to locate a list of Watson's 790. Unfortunately he died in the middle 1980's, well before I got interested. It is sad that Jones and Pall are also gone -- I knew both of them well but I never met Watson or corresponded with him. In 1938 I took a course from Dickson, the founder of the theory of regular ternary forms!

5.
Are you aware that in Watson's first announcement he had 787? Then he changed to 790.

6. Anyway, finding all the regulars was a big job. I have written a document outlining how it was done. I am not really into the electronic world and what I have written (often handwritten) is available only by snailmail. I will be happy to send anything. Is Math Dept Princeton the address to use? (In the AMS Directory there is a Bethpage address and a Harvard email address.}

7. About proofs of regularity. Wi;;'s document contains a lot of them, including some very clever ones. He got very good at it, far outdoing me. I have a handwritten document carrying out the promise to "account for" all 97 proofs. Assembling all the proofs is a bit of a jumble. Two preprints are needed plus many handwritten pages. Watson's unpublished thesis sould also be on hand. You are the first person to express an interest!
8. Did you use Watson's method? Have you worked or do you plan to work on regulars that are not loners?

That's all I can think of.

Sincerely
Kap (usual nickname). Irving Kaplansky kap@msri.org

Date: Tue, 15 Jun 1999 13:17:15 -0400 (EDT)
From: Manjul Bhargava <bhargava@math.Princeton.EDU>
To: Irving Kaplansky <kap@msri.org>
Subject: Re: Welcome to the world of ternary forms

Dear Prof. Jagy and Prof. Kap,

Thanks for your messages! It was so nice to hear from you.

I mean to tell you shortly as to how I got into the world of ternary forms. But in the meantime, I wanted to let you know that I just started going through our lists by hand (my list of loners is on computer, and in a very different notation than yours, so I thought I would just sit in front of the computer and compare our lists). So far the only discrepancy I find is that (in your notation)

512: 1 8 64 0 0 0

is unique in its genus. Please confirm whether this is the case, or whether I may have made a computational error.

Every other way, our lists seem to agree (so far).

Oh yes, please do send me your preprints (or prepreprints :) ) as to proofs of regularity; I am very intrigued that you were able to prove so many forms regular! Thanks-- my address is Dept. of Mathematics, Princeton University, Princeton NJ, 08544.

More soon--

Best regards,
Manjul

Date: Tue, 15 Jun 1999 13:53:04 -0700 (PDT)
From: kap@msri.org (Irving Kaplansky)
To: bhargava@math.princeton.edu, kap@msri.org
Subject: It's not a loner

1 8 64 is not a loner. It is regular with genus mate 4 8 17 0 4 0.

Kap

Date: Tue, 15 Jun 1999 21:15:22 -0400 (EDT)
From: Manjul Bhargava <bhargava@math.Princeton.EDU>
To: Irving Kaplansky <kap@msri.org>
Subject: Re: It's not a loner

Dear Prof. Kaplansky,

Thanks so much for your e-mail. All the other entries agreed! I just worked out the one deviant case by hand, and it seems you're totally right that the form has class number 2. (I wonder what the bug was with my program! (And why would it mess up on just one form??) It's a mystery to me.)

Getting back to what I wanted to write you about earlier-- I happened to be thinking about ternary "loners" because of another problem I was thinking about, which I now realize will also interest you! I just saw your paper on ternary forms representing all odd integers.

Do you know of Conway and Schneeberger's 15 theorem? I recently found a simple proof; essentially, I constructed a list of 11 ternary loners, and showed that _any_ form representing all integers must have one of these 11 loners as a subform. I was thereby able to classify all universal forms, resulting in the 15 theorem.

Having found this proof, I thought it'd be interesting to give a similar classification of forms representing all odd integers. For the time being, I only looked at the integer-matrix (even) case. By finding enough loners (and luckily I didn't use the (1 8 64) form!--oh I realize now it wouldn't have mattered, since you show it is regular anyhow), I was able to show: if a quadratic form (any number of variables) represents 1, 3, 5, 7, 11, 15, and 33, then it represents all odd integers.

Now in your article you also give a (speculated) list of all integer-valued (odd) forms representing all odd integers. Have you been able to prove any of the four leftovers? I notice that one of them has now been proven regular in your subsequent paper. (If the remaining three cases could be proved, we could probably obtain an extension of the 33 theorem, which would contain in particular the entire ternary case.)

Professor Sarnak also sends his regards. (He is the one who mentioned your paper on ternary forms representing odd numbers to me.)

Best regards,
Manjul

Date: Wed, 16 Jun 1999 10:19:12 -0700 (PDT)
From: kap@msri.org (Irving Kaplansky)
To: bhargava@math.princeton.edu, kap@msri.org
Subject: More

Dear Manjul:

Got your latest. Thanks. Indeed I am eager to learn more about how you did your things. Please send whatever can be sent. Let me just repeat one question: on ternary loners did you use Watson's method? If not, how did you prove you have them all?

I have put in the mail a first instalment of what I plan to send you.

On ternaries representing odd numbers, everything is pretty trivial on even ones. On odd ones in my Acta Arithm. paper I should have said that representation of 1, 3, 5, 7 bounds the discriminant by 77 (best possible). This is done by an easy apriori bound method that I learned from Watson, but it may be much older. I have used it again and again; in particular it played a big role in finding the regular ternaries. It is a minor task to examine all ternaries up to discriminant 77; I have a list of all which represent 1,3,5,7.

Yes, Will settled one of the four mysteries in my paper. It's in his Acta Arithm. paper. A copy is on the way to you. I would love to see any or all of the 3 remaining bite the dust. It's beyond me.

In this vein, when I learned from Hsia about Conway's 31 theorem (are you familiar with this?) I did it over again my way. In fact I found all ternaries that represent 1,2,3,5 (a priori bound plus a brutal examination of a handful of forms).

O yes let me express my delight that the 794 loners stood up. Such a confirmation is important. Thanks for doing it.

I'll close this email by passing on to you my favorite quaternary challenge. Does the diagonal form 1,3,5,7 represent everybody except 2 and 22? It's true up to 2 million. I have pestered many people about this (including Conway). On another occasion I'll tell you how this came up.

Kap

Date: Wed, 16 Jun 1999 14:13:08 -0700 (PDT)
From: kap@msri.org (Irving Kaplansky)
To: bhargava@math.princeton.edu, kap@msri.org
Subject: Supplement

You might want to put the following in your file: the ternary 1 1 3 1 0 0 represents all odd numbers through 75 but misses 77 (and then 143, 187, ...).

At the risk of boring you to tears: if a ternary represents all numbers through 77 (of course many could be omitted) then it represents them all or else is one of the three infamous ones whose status is still in doubt. (The three criminals are OK up to 2 million.)

I should have said all odd numbers through 77.

Date: Thu, 17 Jun 1999 18:08:47 -0400 (EDT)
From: Manjul Bhargava <bhargava@math.Princeton.EDU>
To: Irving Kaplansky <kap@msri.org>
Subject: Re: Supplement

Dear Prof. Kap,

Thanks so much for your e-mails. There's quite a bit I wanted to write to you, but for now I will have to leave it at this, and the rest will have to wait till I return next month. I may have sporadic access to e-mail.

Yikes, there is a ternary form representing every odd number up to 75, and missing 77 ! It seems that the n-theorem for integer-valued forms representing all odd numbers will have very large n indeed. (n is only 33 if we are speaking of integer-matrix forms.)

I'm afraid the computation I did shouldn't be considered a reconfirmation of your result. I was simply trying to reproduce your list of loners, rather than reconfirm it. I needed an electronic list, and when I hadn't heard from Prof. Jagy for several days, I decided to write a program that could produce the list for me. I assumed your list was correct, so I found all loners within the bounds evident from your paper! So if for some reason there are some larger loners out there, I too will have missed them. :(

Thanks again for the informative e-mails! In the meantime, I will try and think about your [1,3,5,7] question! (a nice problem!)

Best regards and more soon--
Manjul