1. This was meant to be Ex. XI.6.2 on homework set 6. ◡̈

2. (a) (i) $f(1, S) = 1S1^{-1} = S$
   (ii) $f(\sigma, f(\tau, S)) = f(\sigma, \tau S \tau^{-1}) = \sigma\tau S \tau^{-1}\sigma^{-1} = \sigma\tau S(\sigma\tau)^{-1} = f(\sigma\tau, S)$

   (b) Since $[S_3 : 1] = 6 = 3 \cdot 2$, the 2-groups of $S_3$ are Sylow. They are the subgroups $H, H', H''$ generated by the 3 involutions $(1, 2)$, $(2, 3)$, $(1, 3)$ respectively. Any conjugate of $H$ is a 2-group and conversely by the second Sylow theorem all 2-groups are conjugate, so the orbit of $H$ is $\{H, H', H''\}$.

   Since $[S_3 : A_3] = 2$, $A_3 \triangleleft S_3$, so the orbit of $A_3$ is trivial.

   (c) By the second Sylow theorem the size of the orbit is the index of the normalizer, so $[S_3 : N_{S_3}(H)] = 3$. By Lagrange's theorem $[N_{S_3}(H) : 1] = 2$, but $H < N_{S_3}(H)$, so $H = N_{S_3}(H)$.

   Since $A_3 \triangleleft S_3$, $N_{S_3}(A_3) = S_3$.

3. (a) $\Delta'_n$ is generated by $a^2$ (see Ex. XII.3.4 on homework set 8).

   (b) The subgroup $(a)$ has size 5 and index 2, so is normal (in fact, $(a) = \Delta'_5$). This gives a composition series $\Delta_5 \triangleright (a) \triangleright 1$ with corresponding factors $\mathbf{Z}_2$ and $\mathbf{Z}_5$.

4. (a) By long division $u^2 - 2u + 3 = (u - 1)^2 + 2$. Therefore, $s = \frac{1}{2}u - \frac{1}{2}$.

   (b) Let $w = u - 1$. Then $u = w + 1$, so $0 = (w + 1)^2 - 2(w + 1) + 3 = w^2 + 2$
   Dividing by $w^2$ we get $0 = 1 + 2w^{-2} = 1 + 2s^2$