

① let $m = \text{lcm}(a, b)$

let k be a common multiple of a & b

Div. alg. $\Rightarrow k = qm + r$ for some q, r
 $0 \leq r < m$

Then $r = k - qm$

Since both k & m are common multiples of a & b , so is r

Since m is the least, $r = 0$ $\ddot{\smile}$

Alt. Let p_1, \dots, p_n be all distinct prime divisors of a and b .

Exponential notation $a = p_1^{k_1} \dots p_n^{k_n}$

$b = p_1^{l_1} \dots p_n^{l_n}$ (some powers may be 0)

$\text{lcm}(a, b) = p_1^{\max(k_1, l_1)} \dots p_n^{\max(k_n, l_n)}$

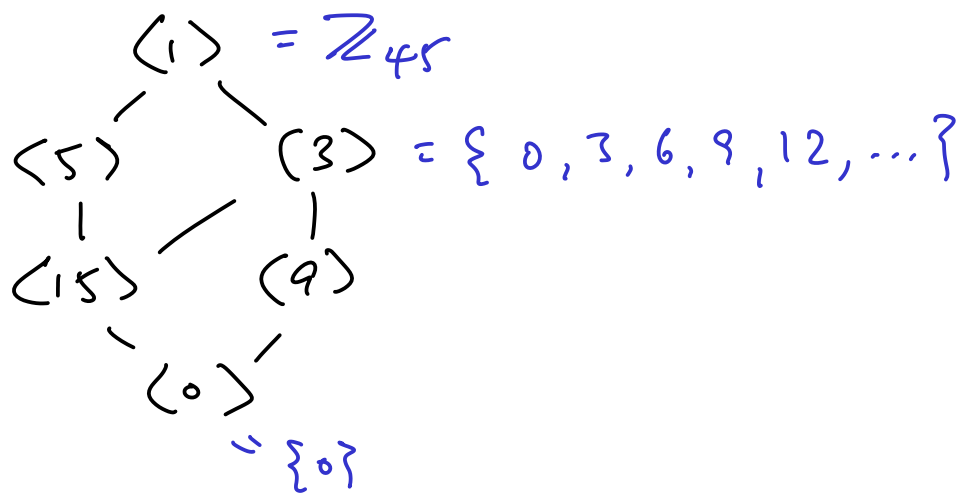
Let k be a common multiple of a & b

Then for each i $p_i^{k_i} \mid k$

and $p_i^{l_i} \mid k$, so $p_i^{\max(k_i, l_i)} \mid k$

$\ddot{\smile}$

(2)



(3)

Let G be a nontrivial finite group and let $a \in G$, $a \neq e$.

Since G is finite $|a| < \infty$

Let $n = |a|$. Since $a \neq e$

$n > 1$, so $\exists p$ prime s.t. $p | n$

Then $\exists m$ $n = pm$

Then $a^n = a^{pm} = (a^m)^p = e$

and that's the smallest pos. power.

$\therefore |a^m| = p$. \smile

(4)

Let $b = a^5$, then

$b^5 = a^{25} = a^{24} \cdot a = e \cdot a = a$ \smile