(1) Let $m = \text{lcm}(a,b)$

Let k be a common multiple of a & b

Div. alg. $\Rightarrow$ $k = qm + r$ for some q, r
$0 \leq r < m$

Then $r = k - qm$
Since both k & m are common multiples of a & b, so is r
Since m is the <u>least</u>, $r = 0$:)

<u>Alt.</u> Let $p_1 \cdots p_n$ be all distinct prime divisors of a and b.

Exponential notation $a = p_1^{k_1} \cdots p_n^{k_n}$
$b = p_1^{l_1} \cdots p_n^{l_n}$ (some powers may be 0)

$\text{lcm}(a,b) = p_1^{\max(k_1, l_1)} \cdots p_n^{\max(k_n, l_n)}$

Let k be a common multiple of a & b
Then for each i $p_i^{k_i} \mid k$
and $p_i^{l_i} \mid k$, so $p_i^{\max(k_i, l_i)} \mid k$

:)

②

$$
\begin{tikzcd}[column sep=small]
 & \langle 1 \rangle = \mathbb{Z}_{45} \arrow[dl, dash] \arrow[dr, dash] & \\
\langle 5 \rangle \arrow[d, dash] & & \langle 3 \rangle = \{0, 3, 6, 9, 12, \dots\} \arrow[d, dash] \arrow[dll, dash] \\
\langle 15 \rangle \arrow[dr, dash] & & \langle 9 \rangle \arrow[dl, dash] \\
 & \langle 0 \rangle = \{0\} &
\end{tikzcd}
$$

③ Let G be a nontrivial finite group and let $a \in G$, $a \neq e$.

Since G is finite $|a| < \infty$

Let $n = |a|$. Since $a \neq e$

$n > 1$, so $\exists$ p prime s.t. $p \mid n$

Then $\exists m$ $\quad n = pm$

Then $\quad a^n = a^{pm} = (a^m)^p = e$

and that's the smallest pos. power.

$\therefore \quad |a^m| = p$. ☺

④ Let $b = a^5$, then

$b^5 = a^{25} = a^{24} \cdot a = e \cdot a = a$ ☺