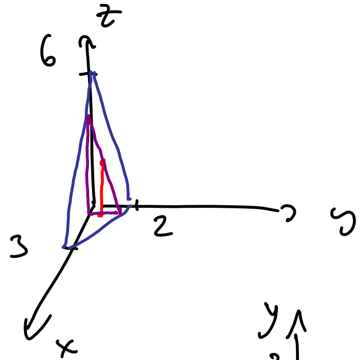
Note Title 11/25/2008

F 2004 Mid 2

①

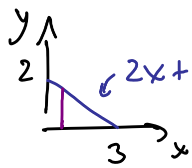


$2x + 3y + z = 6$

density $= 10 + x + y$

z, y, x

$$\int_0^3 \left[\int_0^{2-\frac{2}{3}x} \left[\int_0^{6-2x-3y} (10+x+y)\, dz \right] dy \right] dx$$



$2x+3y=6$

$y = \frac{6-2x}{3} = 2 - \frac{2}{3}x$

②

$(1,1) \to (5,3)$ $\int y\, dx$

Parametrize $\bar{x} = \begin{bmatrix}1\\1\end{bmatrix}(1-t) + \begin{bmatrix}5\\3\end{bmatrix} t$ $0 \le t \le 1$

(weighted average)

$\bar{X} = \begin{bmatrix}1+4t\\1+2t\end{bmatrix}$ ← y $d\bar{X} = \begin{bmatrix}4\\2\end{bmatrix} dt$ dx

↖ direction vector $\left(\begin{bmatrix}5\\3\end{bmatrix} - \begin{bmatrix}1\\1\end{bmatrix} \right)$

$\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}4\\2\end{bmatrix} t$

("u + tv" formula)

$$\int y\, dx = 4 \int_0^1 (1+2t)\, dt = 4\left(t + t^2\right)\Big|_0^1 = \boxed{8}$$

$$\parallel$$

$\int \bar{F} \cdot d\bar{x}$, where $\bar{F} = [y, 0]$

$d(y\, dx) = dy\, dx = -dx\, dy \neq 0$

so integrals are found to be path dependent

(3) Surface $[s^2t, st^2, s+t]$ @ $[-4, 2, 1]$

Parametric formula: use linear approximation:

$a = [2,-1]$

$$f(x) \approx f(a) + Df(a)(x-a)$$

For which s,t is the surface @ $[-4,2,1]$

Set
$$\left.\begin{aligned} s^2t &= -4 \\ st^2 &= 2 \end{aligned}\right) \quad \frac{s}{t} = -2$$
$$s+t = 1 \qquad s=2, \; t=-1$$

$$D\underline{X} = \begin{bmatrix} 2st & s^2 \\ t^2 & 2st \\ 1 & 1 \end{bmatrix}$$

$$X(s,t) = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ 1 & -4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s-2 \\ t+1 \end{bmatrix}$$

T_s T_t

$$= \begin{bmatrix} -4 - 4(s-2) + 4(t+1) \\ 2 + s-2 - 4(t+1) \\ 1 + s-2 + t+1 \end{bmatrix} = \ldots$$

Since we are in $\mathbb{R}^3$ a plane can be specified by an equation.

$$N = T_s \times T_t = [5, 8, 12]$$

$$\boxed{5(x+4) + 8(y-2) + 12(z-1) = 0}$$

(4) $F = [\,(x-1)^2 y^2,\ y,\ z\,]$ unit disc in the y-z plane

$$X(r,\theta) = \begin{bmatrix} 0 \\ r\cos\theta \\ r\sin\theta \end{bmatrix} \qquad \begin{matrix} 0 \le r \le 1 \\ -\pi < \theta \le \pi \end{matrix}$$

$$dX = \begin{bmatrix} 0 \\ dr\cos\theta - r\sin\theta\, d\theta \\ dr\sin\theta + r\cos\theta\, d\theta \end{bmatrix}$$

$\perp$ to the disc :)

$$dS = \begin{bmatrix} dy\,dz \\ dz\,dx \\ dx\,dy \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} dr\,d\theta$$

$$\int F\cdot dS = \int_{-\pi}^{\pi}\int_0^1 r^2(\cos\theta)^2\, r\, dr\, d\theta$$

On average $(\cos\theta)^2 = \frac{1}{2}$

$$= \underbrace{\int_{-\pi}^{\pi}(\cos\theta)^2 d\theta}_{\frac{1}{2}\cdot 2\pi \,=\, \pi}\ \underbrace{\int_0^1 r^3\, dr}_{1/4} = \boxed{\frac{\pi}{4}}$$

$$(\cos\theta)^2 = \frac{1+\cos(2\theta)}{2}$$

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(5) $\omega = e^{xy}$ $\qquad \eta = x\,dy + y\,dz$

$d\omega\wedge\eta$? $\qquad d\omega\wedge d\eta$?

$$d\omega = e^{xy}\, d(xy) = e^{xy}(dx\cdot y + x\cdot dy)$$
$$= \underbrace{ye^{xy}}_{\frac{\partial\omega}{\partial x}} dx + \underbrace{xe^{xy}}_{\frac{\partial\omega}{\partial y}} dy$$

$$d\eta = dx\,dy + dy\,dz = dy\,dz + dx\,dy$$

$$d\omega\wedge\eta = (ye^{xy}\,dx + xe^{xy}\,dy)(x\,dy + y\,dz)$$

$$= \underline{xy e^{xy}\, dy\, dz - y^2 e^{xy}\, dz\, dx + xy e^{xy}\, dx\, dy}$$

$$d\omega \wedge d\eta = \left(y e^{xy}\, dx + x e^{xy}\, dy\right)\left(dy\, dz + dx\, dy\right)$$

$$= \underline{y e^{xy}\, dx\, dy\, dz}$$