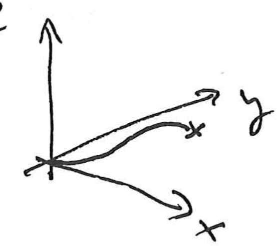
MAT 3243 Midterm I Fall 2001

Dg

① a) To find target set $z(t)=0$ and solve for t.

$$3t-t^2=0$$



$$t(3-t)=0$$

$t=0$ or $t=3$ min

↑ Launch ↑ Target.

$x(3)=3$ km, $y(3)=9$ km ⇐ Target.

Distance to target $=\sqrt{3^2+9^2}=\sqrt{90}\approx 10$ km.

b) Velocity : $\begin{bmatrix} x'(t) \\ y'(t) \\ z'(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 2t \\ 3-2t \end{bmatrix}$

at target $\begin{bmatrix} x'(3) \\ y'(3) \\ z'(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ -3 \end{bmatrix}$ km/min

Speed $=\sqrt{1^2+6^2+3^2}=\sqrt{46}\approx 7$ km/min

(2) $$df = \begin{bmatrix} 2\,dx + 2y\,dy \\ 2x\,dx \cdot y + x^2\,dy \end{bmatrix}$$

$$\therefore Df = \begin{bmatrix} 2 & 2y \\ 2xy & x^2 \end{bmatrix}$$

$$Df(p) = \begin{bmatrix} 2 & 2\cdot 2 \\ 2\cdot 1\cdot 2 & 1^2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 1 \end{bmatrix}$$

Pt-slope: $\Delta f \approx Df(p)\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} 2\Delta x + 4\Delta y \\ 4\Delta x + \Delta y \end{bmatrix}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} \qquad f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2\cdot 1 + 2^2 \\ 1^2\cdot 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Linear approximation:

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \approx \begin{bmatrix} 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 2(x-1) + 4(y-2) \\ 4(x-1) + y - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2x + 4y - 4 \\ 4x + y - 4 \end{bmatrix}$$

(3) a) $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial f}{\partial t}\dfrac{\partial t}{\partial x}$

$\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial s}\dfrac{\partial s}{\partial y} + \dfrac{\partial f}{\partial t}\dfrac{\partial t}{\partial y}$

$$\begin{bmatrix} \dfrac{\partial s}{\partial x} & \dfrac{\partial s}{\partial y} \\ \dfrac{\partial t}{\partial x} & \dfrac{\partial t}{\partial y} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$$

$\dfrac{\partial f}{\partial x} = 3\dfrac{\partial f}{\partial s} + \dfrac{\partial f}{\partial t}$

$\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial s} - \dfrac{\partial f}{\partial t}$

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b) $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial\left[\frac{\partial f}{\partial y}\right]}{\partial x} = \dfrac{\partial\left[\frac{\partial f}{\partial s} - \frac{\partial f}{\partial t}\right]}{\partial x}$

$= 3\dfrac{\partial\left(\frac{\partial f}{\partial s} - \frac{\partial f}{\partial t}\right)}{\partial s} + \dfrac{\partial\left(\frac{\partial f}{\partial s} - \frac{\partial f}{\partial t}\right)}{\partial t}$

$= 3\dfrac{\partial^2 f}{\partial s^2} - 3\dfrac{\partial^2 f}{\partial s \partial t} + \dfrac{\partial^2 f}{\partial t \partial s} - \dfrac{\partial^2 f}{\partial t^2}$

$= 3\dfrac{\partial^2 f}{\partial s^2} - 2\dfrac{\partial^2 f}{\partial s \partial t} - \dfrac{\partial^2 f}{\partial t^2}$

4) a) $\int_1^2 \int_0^1 x^2 \log y \; dx\, dy$

$= \int_1^2 \log y \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \int_1^2 \log y \; dy$

$= \frac{1}{3} \left[y \log y - \int y \, d(\log y) \right]_1^2$

$= \frac{1}{3} \left[y \log y - \int y \frac{1}{y} dy \right]_1^2$

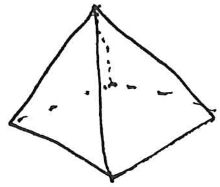
$= \frac{1}{3} \left[y \log y - y \right]_1^2 =$

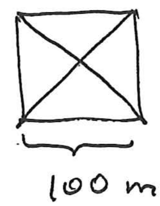
$= \frac{1}{3} \left[2 \log 2 - 2 - (1 \cdot \underbrace{\log 1}_{0} - 1) \right]$

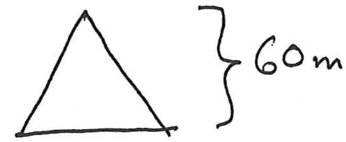
$= \frac{1}{3} \left[2 \log 2 - 1 \right]$

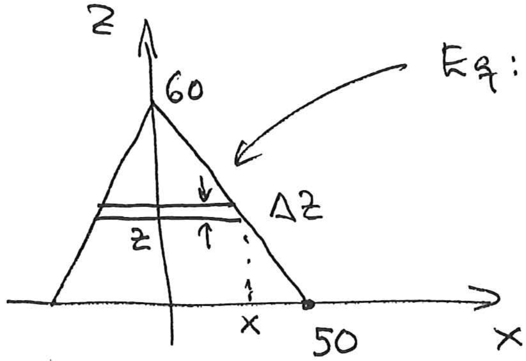
b)

Top: Side:









Eq: Slope $= -\frac{60}{50} = -\frac{6}{5}$

so $z = -\frac{6}{5}x + 60$

so $x = -\frac{5}{6}z + 50$

Volume of each slab: $(2x)^2 \cdot \Delta z$

$= \left(-\frac{5}{3}z + 100\right)^2 \Delta z$

Total volume: $\sum \left(-\frac{5}{3}z+100\right)^2 \Delta z$

$$\Big\downarrow \lim_{\Delta z \to 0}$$

$$\int_0^{60} \left(-\frac{5}{3}z+100\right)^2 dz$$

$$= \frac{\left(-\frac{5}{3}z+100\right)^3}{\cancel{3}} \cdot \left(-\frac{\cancel{3}}{5}\right)\Bigg|_0^{60} = \left(-\frac{1}{5}\right)\left[\left(-\frac{5}{3}\cdot 60+100\right)^3 - 100^3\right]$$

$$= \left(-\frac{1}{5}\right)\left(-100^3\right) = \frac{1000000}{5} = 200000 \text{ m}^3$$