1. If P, Q, R are propositions, use a truth table to prove that $(P \wedge Q) \vee R \Leftrightarrow (P \vee R) \wedge (Q \vee R)$

   The outputs in columns 5 and 8 below are the same.

p	q	r	p ∧ q	p ∧ q ∨ r	p ∨ r	q ∨ r	(p ∨ r) ∧ (q ∨ r)
true	true	true	true	true	true	true	true
true	true	false	true	true	true	true	true
true	false	true	false	true	true	true	true
true	false	false	false	false	true	false	false
false	true	true	false	true	true	true	true
false	true	false	false	false	false	true	false
false	false	true	false	true	true	true	true
false	false	false	false	false	false	false	false

2. Using formal language and appropriate quantifiers, translate into symbolic form the following sentences. Determine whether they are equivalent and explain why or why not.

   - Every integer is even or odd.
     $\forall n \in \mathbf{Z}$ (n is even $\vee$ n is odd)
   - Every integer is even or every integer is odd.
     $(\forall n \in \mathbf{Z}$ (n is even)) $\vee$ ($\forall n \in \mathbf{Z}$ (n is odd))

   The two sentences are not equivalent. The first sentence is true, but the second is false. There are integers that are not even (for example 1) and there are integers that are not odd (for example 0) ⍨

3. An integer greater than 1 is called prime precisely when its only positive divisors are itself and 1. Write out this statement in the language of formal logic using appropriate quantifiers. Then negate it (and simplify) and write out the negation in words.

   $n \in \mathbf{Z}$ is prime means $n > 1 \wedge \forall d \in \mathbf{Z}\ [(d > 0 \wedge d|n) \Rightarrow (d = 1 \vee d = n)]$

   Negation: $n \leq 1 \vee \exists d \in \mathbf{Z}\ (d > 0 \wedge d|n \wedge d \neq 1 \wedge d \neq n)$

   $n \in \mathbf{Z}$ is not prime means $n \leq 1$ or n is composite, meaning there is a positive divisor of n that is non-trivial (neither 1 nor n)

4. Suppose A, B, C are sets. For each of the following statements determine whether it is true. If true, prove it. If not, provide a concrete counterexample and explain why it works.

   (a) $(A \subseteq B \wedge B \subseteq C) \Rightarrow A \subseteq C$

   True.

   Assume $A \subseteq B \wedge B \subseteq C$ and let $x \in A$
   Since $A \subseteq B$, $x \in B$ so since $B \subseteq C$, $x \in C$ $\smile$

   (b) $(A \subseteq B \wedge B \not\subseteq C) \Rightarrow A \not\subseteq C$

   Sometimes false.

   For example, let $A = \varnothing, B = \{0\}, C = \varnothing$
   Then $A \subseteq B \wedge B \not\subseteq C \wedge A \subseteq C$ $\frown$