Midterm 1

1. $(p \to q) \vee (q \to p)$

p	q	$p \to q$	$q \to p$	$(p\to q)\vee(q \to p)$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	T
T	T	T	T	T

$\therefore (p \to q) \vee (q \to p)$ is a tautology

(2) $m \mid n$ means $\exists k \in \mathbb{Z} \quad n = mk$

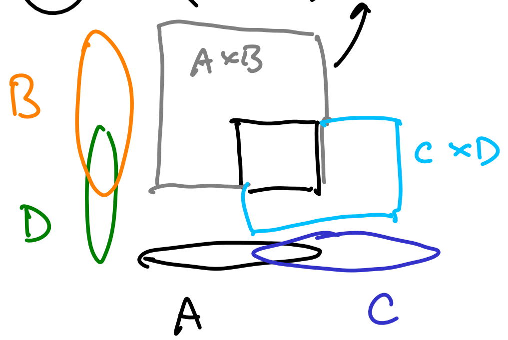
$m \nmid n$ means $\forall k \in \mathbb{Z} \quad n \neq mk$

(3) n is a prime means $n \geq 2 \wedge \forall d \in \mathbb{N} \left[d \mid n \to (d=1 \vee d=n) \right]$

n is not prime means $\underset{n<2}{n=1} \vee \exists d \in \mathbb{N} \left[d \mid n \wedge \neg(d=1 \vee d=n) \right]$

i.e. $n=1 \vee \exists d \in \mathbb{N} \left[d \mid n \wedge d \neq 1 \wedge d \neq n \right]$

(4) $(A \cap C) \times (B \cap D) \ \overset{?}{} \ (A \times B) \cap (C \times D)$



Claim:

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

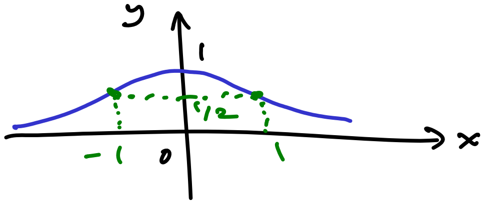
pf $[x,y] \in (A \cap C) \times (B \cap D)$

$\Leftrightarrow x \in A \cap C \ \wedge \ y \in B \cap D$

$\Leftrightarrow x \in A \wedge x \in C \wedge y \in B \wedge y \in D$

$\Leftrightarrow [x,y] \in A \times B \ \wedge \ [x,y] \in C \times D \Leftrightarrow [x,y] \in$ r.h.s. ◡̈

⑤ $f: \mathbb{R} \to \mathbb{R} \qquad f(x) = \dfrac{1}{1+x^2}$



a) $f_*(\mathbb{R}) = (0,1] = \{y \in \mathbb{R} : y > 0 \wedge y \leq 1\}$

b) $f^*(\mathbb{R}) = \mathbb{R}$

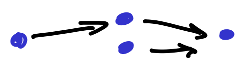
c) $f^*(\{\frac{1}{2}\}) = \{-1, 1\}$

Set $\dfrac{1}{1+x^2} = \dfrac{1}{2}$, solve: $1+x^2 = 2$, $x^2 = 1$, $x = 1 \vee x = -1$

d) Let $A = (-\infty, 0] \cup (1, \infty) = \{x \in \mathbb{R} : x \leq 0 \vee x > 1\}$

Then $A \neq \phi$, but $f^*(A) = \phi$

⑥



Let $X = \{x\} = Z$

Let $Y = \{y, y'\}$ where $y \neq y'$

Let $f: X \to Y$ be defined by $f(x) = y$

Let $g: Y \to Z$ be defined by $g(y) = g(y') = x$

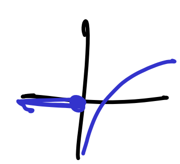
By inspection f is not onto & g is not 1-1

But $g \circ f$ is a bijection.

Another example: $[0,\infty) \xrightarrow{\text{square root}} \mathbb{R} \xrightarrow{\text{square}} [0,\infty)$

composition = identity on $[0,\infty)$

<u>Extra Credit</u> $\qquad \mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{g} \mathbb{R}$



$f(x) = e^x \qquad g(x) = \begin{cases} \ln x & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

$(g \circ f)(x) = g(f(x)) = g(e^x) = \ln(e^x) = x$