Midterm 2 Fall 2012 MAT 3013

1. F is injective:

Let $x, x' \in \mathbb{R}$ s.t. $F(x) = F(x')$

$$[x, f(x)] = [x', f(x')]$$

$$x = x' \quad \ddot\smile$$

F is not onto:

$[x, f(x)+1]$ is not in the range of F

Partial inverse:

Define $G : \mathbb{R}^2 \to \mathbb{R}$ by $G([x,y]) = x$

$$(G \circ F)(x) = G(F(x)) = G([x, f(x)])$$

$$= x \quad \ddot\smile$$

Define $G'([x,y]) = \begin{cases} x & \text{if } y = f(x) \\ 0 & \text{otherwise} \end{cases}$

(2) Reflexive: if $x \in X$, then $f(x) = f(x)$

so $x \sim x$

Symmetric: Suppose $x \sim x'$

then $f(x) = f(x')$, so $f(x') = f(x)$

so $x' \sim x$

Transitive: Suppose $x \sim x'$, $x' \sim x''$

Then $f(x) = f(x')$ and $f(x') = f(x'')$

So $f(x) = f(x'')$, so $x \sim x''$

If f is injective

$x \sim x' \Rightarrow f(x) = f(x') \Rightarrow x = x'$

so eq. classes are singletons.

If f is const. for any x, x'

$f(x) = f(x')$, so $x \sim x'$

So the only eq. class is all of X.

If $f : \mathbb{R} \to \mathbb{R}$ $f(x) = x^2$

if $x \sim x'$ then $f(x) = f(x')$, so $x^2 = x'^2$

so $x = \pm x'$, so eq. classes are $\{0\}$ and

pairs $\{x, -x\}$

(3) Induction on the size

Basis: size $= 1$

If $A = \{a\}$, then $a = \min A$

Inductive step: size > 1

Assume any set of smaller size has a min.

Since $A \neq \phi$ $\exists a \in A$

$A \setminus \{a\}$ has smaller size than A

So $\exists b = \min(A \setminus \{a\})$

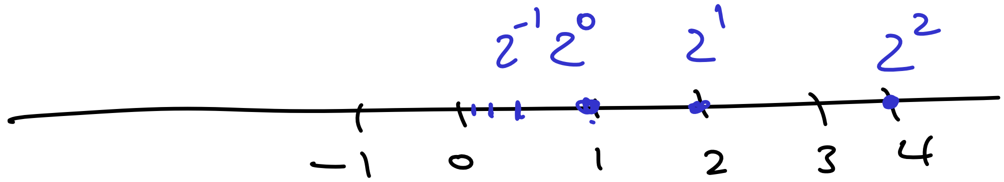
Since A is linearly ordered,

$a \leq b \quad \vee \quad b \leq a$

$\downarrow \qquad\qquad \downarrow$

$a = \min A \qquad b = \min A$

(4)



No max or min. $\sup = \infty$

$\inf = 0$

<u>Min</u>: If $x \in S$, then $\exists n$ $x = 2^n$

Then $2^{n-1} < 2^n \wedge 2^{n-1} \in S$

so x is not a min.

<u>Inf</u> $\forall n$ $2^n > 0$, so 0 is a lower bound

Let $u > 0$. Let $n < \log_2 u$

Then $2^n < u$, so u is not a lower bound.

(5) Let $\{A_i\}_{i\in I}$ be a collection of initial segments indexed by a set I.

Let $a \in \bigcup_{i\in I} A_i$ and let $b < a$

Then $\exists\, j \in I$ $a \in A_j$.

Since A_j is an initial segment, $b \in A_j$

so $b \in \bigcup_{i \in I} A_i$.

Claim: Union of <u>all</u> Dedekind cuts is $\mathbb{Q}$.

Let $a \in \mathbb{Q}$, then $a \in \underbrace{\{ b \in \mathbb{Q} : b < a+1 \}}_{\text{Dedekind cut}}$

so $a \in$ union.