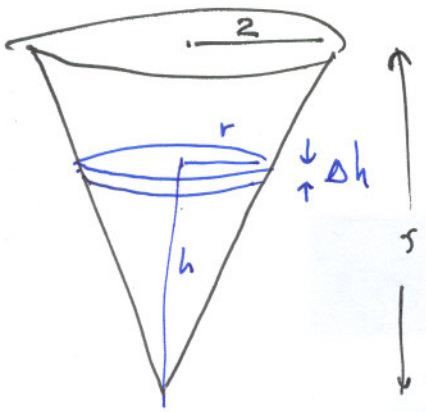
Honors Cal II Sp. 2006 Midterm 2

(1)



$$\frac{r}{h} = \frac{2}{5}$$

$$r = \frac{2}{5}h$$

$$\Delta V = \pi r^2 \Delta h$$

$$= \pi \left(\frac{2}{5}h\right)^2 \Delta h$$

$$= \frac{4\pi}{25} h^2 \Delta h$$

$$\Delta m = \delta \Delta V = \frac{4\pi}{25} 10^3 h^2 \Delta h$$

$$\Delta W = \Delta m \, g \, (5-h) = \frac{4\pi}{25} 10^3 h^2 g (5-h) \Delta h$$

$$W = \frac{4\pi}{25} 10^3 g \int_0^5 h^2 (5-h) \, dh = 2.568 \cdot 10^5$$

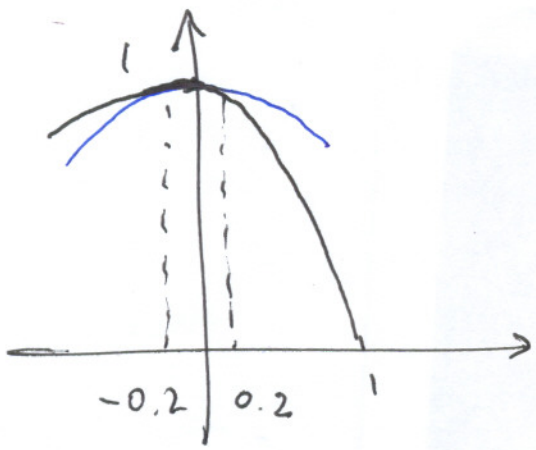
∴ the work is $\boxed{2.568 \cdot 10^5 \text{ joules}}$.

(2)

$$e^x = 1 + x + \frac{x^2}{2} + \dots$$

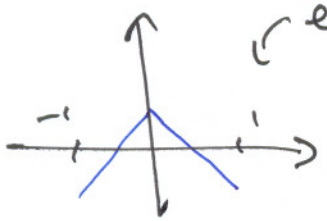
$$e^x(1-x) = 1 + x + \frac{x^2}{2} + \dots - x - x^2 - \frac{x^3}{2} \dots$$

$$= 1 - \frac{x^2}{2} + \dots$$



Good on $[-0.2, 0.2]$
(approx.)

(3)



period $p = 2$

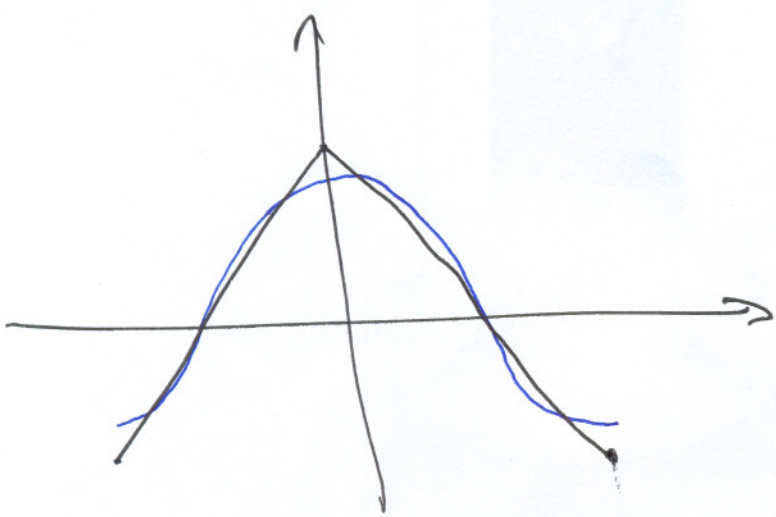
By inspection $a_0 = 0$ and $b_1 = 0$

$$a_1 = \frac{2}{2}\int_{-1}^{1} f(x)\cos\left(\frac{2\pi x}{2}\right)dx$$

$$= \int_{-1}^{1}\underbrace{\left(\tfrac{1}{2} - |x|\right)}_{\text{even}}\cos(\pi x)\,dx = 2\int_{0}^{\frac{1}{2}}\left(\tfrac{1}{2} - x\right)\cos(\pi x)\,dx =$$

$$= \frac{4}{\pi^2} \approx 0.4$$

$$\therefore \quad \frac{1}{2} - |x| \approx 0.4\cos(\pi x)$$



The approximation is poor at $x = -1, 0, 1$

Because f is not differentiable there.

(whereas the Fourier approximation must be differentiable)

④ $\frac{dy}{dx} = (x+1)(y^2+1)$

$$\int \frac{dy}{y^2+1} = \int (x+1)\,dx$$

$$\arctan y = \frac{x^2}{2} + x + C$$

$$y = \tan\left(\frac{x^2}{2} + x + C\right)$$

Since $y(0) = 1$

$1 = \tan(C)$ so $C = \frac{\pi}{4}$

so $y = \tan\left(\frac{x^2}{2} + x + \frac{\pi}{4}\right)$

so $\boxed{y(0.5) = 6.18}$