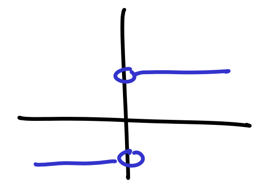
1 a. $f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$



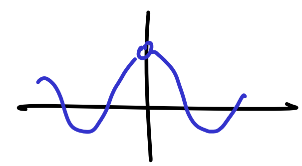
If $x > 0$ $|x| = x$, so $f(x) = \frac{x}{x} = 1$

If $x < 0$ $|x| = -x$, so $f(x) = \frac{-x}{x} = -1$

$\lim_{x \to 0^-} f(x) = -1 \neq \lim_{x \to 0^+} f(x) = 1$

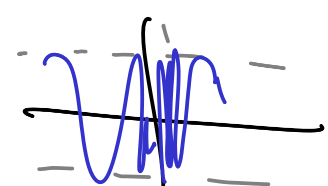
$\therefore \lim_{x \to 0} f(x)$ D.N.E. $\therefore$ No a make f cont.

b) $f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$



$\lim_{x \to 0} \frac{\sin x}{x} = 1$ so if $a = 1$, f is cont.

c) $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$

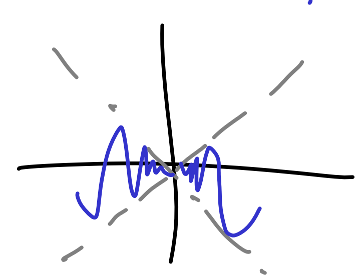


$\lim_{x \to 0} f(x)$ DNE

$\therefore$ No a makes f cont.

If $\delta > 0$
$\sin\left(\frac{1}{x}\right)$ takes on all values between -1 and 1 on the interval $(0, \delta)$

d) $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$



Sandwich $-1 \leq \sin(\ldots) \leq 1$

$-x \leq x \sin(\ldots) \leq x$ (or $\geq$ if $x < 0$)

$\to 0$

$\therefore \lim_{x \to 0} f(x) = 0$ $\therefore a = 0$ makes f cont.

② $y = x^{-1} = \frac{1}{x}$ $\qquad [2, \frac{1}{2}]$

$$y' = \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

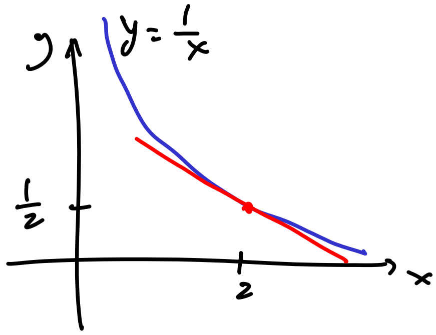
$$= \lim_{h\to 0} \frac{\cancel{x} - (\cancel{x} + h)}{h(x+h)x} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x+h)x} =$$

$$= \lim_{h\to 0} \frac{-1}{(x+h)x} = -\frac{1}{x^2} \qquad \left(\text{Check: } (x^{-1})' = (-1)\,x^{-1-1}\right)$$

Tangent: $y = \underset{\uparrow\ f(x_0)}{y_0} + \underset{\uparrow\ f'(x_0)}{m}(x - x_0)$

$$y = \frac{1}{2} - \frac{1}{2^2}(x-2)$$

$$y = \frac{1}{2} - \frac{1}{4}(x-2)$$



③ a) $\left(\sqrt{x}\ln x\right)' = \left(\sqrt{x}\right)' \ln x + \sqrt{x}\,(\ln x)'$

$$\left(\sqrt{x}\right)' = \left(x^{1/2}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \ln x + \sqrt{x} \cdot \frac{1}{x}$$

$$= \frac{1}{2\sqrt{x}} \ln x + \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}\left[\frac{\ln x}{2} + 1\right]$$

b) $\left(\frac{x}{1+x^2}\right)' = \frac{x'(1+x^2) - x(1+x^2)'}{(1+x^2)^2} = \frac{1 + x^2 - x \cdot 2x}{(1+x^2)^2}$

$$= \frac{1-x^2}{(1+x^2)^2}$$

c) $(\cos(e^x))' = -\sin(e^x)(e^x)' = -\sin(e^x)\cdot e^x$

d) $(\sin(x^e))' = \cos(x^e)(x^e)' = \cos(x^e)e x^{e-1}$

(4) $[f(x^2)]' = f'(x^2)\cdot 2x$

at $x=0$: $f'(0)\cdot 0 = 0$

$[x^2 f(x)]' = 2x f(x) + x^2 f'(x)$

at $x=0$: $2\cdot 0\cdot f(0) + 0^2\cdot f'(0) = 0$

(5) $y = x^2$ $[-3, 8]$

Tangent line: $y = \underset{f(x_0)}{y_0} + \underset{f'(x_0)}{m}(x - x_0)$

$y = x_0^2 + 2x_0(x - x_0)$ $\frac{dy}{dx} = 2x$

if the tangent line passes through $[-3, 8]$,

$8 = x_0^2 + 2x_0(-3 - x_0)$

$8 = x_0^2 - 6x_0 - 2x_0^2$

$8 = -x_0^2 - 6x_0$

$x_0^2 + 6x_0 + 8 = 0$

$x_0 = -2, -4$

Tangent lines:

$y = 4 - 4(x+2)$

$y = 16 - 8(x+4)$