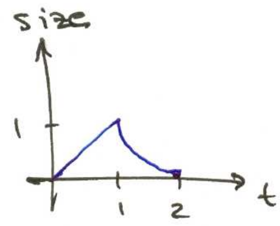
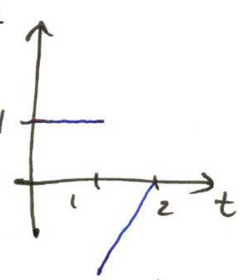
① 



continuous ↑ discontinuous at $t=1$.

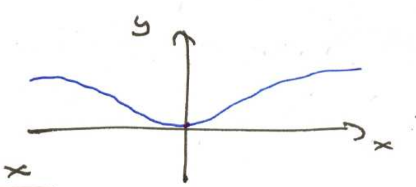
② a) $(\sqrt{x})' = \lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \dfrac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$

$= \lim_{h\to 0} \dfrac{1}{\sqrt{x+h}+\sqrt{x}} = \boxed{\dfrac{1}{2\sqrt{x}}}$

b) $\left(2^{\cos x}\right)' = \ln 2 \cdot 2^{\cos x}(-\sin x)$

c) $(x\tan x)' = \tan x + \dfrac{x}{(\cos x)^2}$

③ $y = \arctan(x^2)$



$y' = \dfrac{1}{1+(x^2)^2}\cdot 2x = \dfrac{2x}{1+x^4}$

$y'' = \dfrac{2(1+x^4)-2x\cdot 4x^3}{(1+x^4)^2} = \dfrac{2+2x^4-8x^4}{(1+x^4)^2}$

$= \dfrac{2(1-3x^4)}{(1+x^4)^2} < 0$ when $x^4 > \frac{1}{3}$

i.e. $\boxed{x > \frac{1}{\sqrt[4]{3}} \text{ or } x < -\frac{1}{\sqrt[4]{3}} \quad \approx 0.76}$

④ Let $f(x) = x\,g(x)$

then $f'(x) = g(x) + x g'(x)$, so $f(0)=0$, $f'(0)=2$

Tangent line: $y = f(0) + f'(0)\,x = 0 + 2x = 2x$

$$\boxed{y = 2x}$$

⑤ a) $\lim\limits_{t\to\infty} t \sin\left(\frac{1}{t}\right) = \lim\limits_{u\to 0} \frac{1}{u}\sin(u) = 1$

$\therefore \lim\limits_{t\to\infty} p(t) = \lim\limits_{t\to\infty} 15 + t\sin\left(\frac{1}{t}\right) = \boxed{16}$

b) $\lim\limits_{t\to\infty} \frac{\sin(t)}{t} = 0$ Because $-1 \le \sin(t) \le 1$

so $-\frac{1}{t} \le \frac{\sin(t)}{t} \le \frac{1}{t}$ (sandwich rule) a.k.a. squeeze law

0

$\therefore \lim\limits_{t\to\infty} p(t) = \lim\limits_{t\to\infty} 15 + \frac{\sin(t)}{t} = \boxed{15}$

⑥ $y\,e^{\cosh x} = x \ln y$

d

$dy \cdot e^{\cosh x} + y\,e^{\cosh x} \sinh x\,dx$

$= dx \cdot \ln y + x\,\frac{1}{y}\,dy$

$\left(e^{\cosh x} - \frac{x}{y}\right) dy = \left(\ln y - y\,e^{\cosh x}\sinh x\right) dx$

$$\therefore \boxed{\frac{dy}{dx} = \frac{\ln y - y\,e^{\cosh x}\sinh x}{e^{\cosh x} - \frac{x}{y}}}$$

(7) objective: minimize distance to the origin

This is equiv. to minimizing its square:

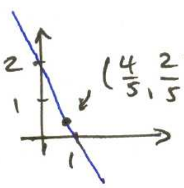
$f(x) = d^2 = x^2 + y^2 = x^2 + (2-2x)^2$

$f'(x) = 2x + 2(2-2x)(-2)$

$= 2(x + 4x - 4) = 2(5x-4)$

$f'(x) = 0$ when $x = \frac{4}{5}$ so $y = 2 - 2x$

$= 2 - \frac{8}{5} = \frac{2}{5}$



$\therefore$ the closest point is $\boxed{\left(\frac{4}{5}, \frac{2}{5}\right)}$

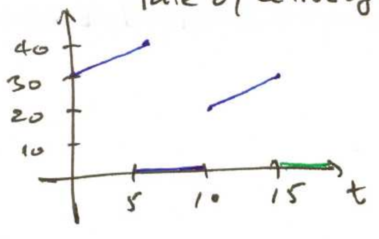
(8)

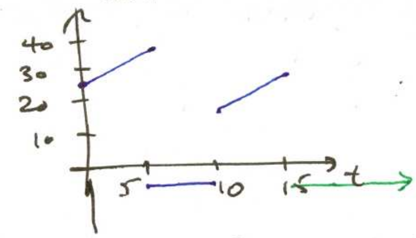
$V = \frac{1}{2}\frac{4}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{\pi}{12} D^3$

$V' = \frac{\pi}{12} 3D^2 D' = \frac{\pi}{4} D^2 D'$

$D' = \frac{4}{\pi D^2} V' = \frac{4}{\pi 0.5^2} \cdot 1 = \frac{4}{0.25\pi} \approx \boxed{5.1 \frac{cm}{year}}$

(9) rate of delivery with clearance





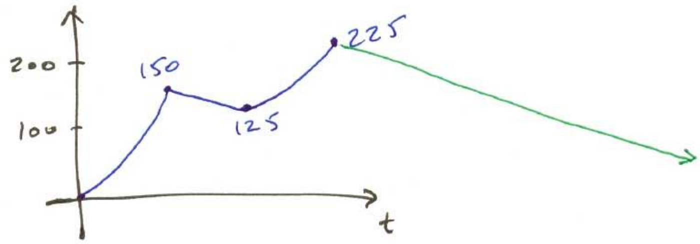
Total area: $\frac{25+35}{2}\cdot 5 - 5\cdot 5 + \frac{15+25}{2}\cdot 5 = 225$

$\therefore$ Accumulation at the end of treatment is $\boxed{225 \text{ mg}}$

This will clear $\boxed{45 \text{ mins}}$ $\left(\frac{225}{5}\right)$ after treatment stops.

Concentration has a local max in 5 mins (150 mg)

But the global max is at the end of treatment



(10) a) $\displaystyle\int_1^4 \frac{(1+\sqrt{t})^2}{t}\,dt = \int_1^4 \frac{1+2\sqrt{t}+t}{t}\,dt$

$\displaystyle = \int_1^4 \left(t^{-1} + 2t^{-\frac{1}{2}} + 1\right)dt = \left[\ln t + 4t^{\frac{1}{2}} + t\right]_1^4$

$= \ln 4 + 4\cdot 2 + \cancel{4} - 0 - \cancel{4} - 1 = \ln 4 + 7 = \boxed{8.386}$

b) $\displaystyle\int_0^1 \cos\left(\frac{\pi t}{4}\right)dt = \frac{4}{\pi}\sin\left(\frac{\pi t}{4}\right)\Big|_0^1 = \frac{4}{\pi}\left(\sin\left(\frac{\pi}{4}\right) - 0\right)$

$= \frac{4}{\pi}\cdot\frac{1}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi} \approx \boxed{0.9}$