MAT 1214 Midterm 3 Fall 2001

(1) a) $\int_1^2 (x^2+1)\,dx = \frac{x^3}{3} + x \Big|_1^2$

$= \frac{2^3}{3} + 2 - \frac{1^3}{3} - 1 = \frac{8}{3} - \frac{1}{3} + 1 = \frac{8-1+3}{3} = \boxed{\frac{10}{3}}$

b) $\int_0^1 \sqrt{2x+1}\,dx = \int_0^1 (2x+1)^{1/2}\,dx = \frac{(2x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \cdot \frac{1}{2} \Big|_0^1$

$\uparrow$ Chain Rule

$= (2x+1)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{2} \Big|_0^1 = \frac{1}{3}(2x+1)^{3/2} \Big|_0^1$

$= \frac{1}{3}\left[(2+1)^{3/2} - (0+1)^{3/2}\right] = \frac{1}{3}\left[3^{3/2} - 1\right]$

$= 3^{\frac{3}{2}-1} - 3^{-1} = 3^{\frac{1}{2}} - 3^{-1} = \boxed{\sqrt{3} - \frac{1}{3}} \approx 1.3987$

c) $\int_0^3 |x-1|^3\,dx = -\int_0^1 (x-1)^3\,dx + \int_1^3 (x-1)^3\,dx$

$|x-1| = \begin{cases} x-1 & \text{for } x \geq 1 \\ -(x-1) & \text{for } x < 1 \end{cases}$

$= -\frac{(x-1)^4}{4}\Big|_0^1 + \frac{(x-1)^4}{4}\Big|_1^3 = \frac{1}{4} + \frac{2^4}{4} = \boxed{\frac{17}{4}}$

d) $\frac{d}{dx}\int_1^x \sqrt{3+\sin(t)}\,dt = \sqrt{3+\sin(x)}$ since $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ (F.T.C.)

e) $\frac{d}{dx}\int_{x^2}^{x^3} \sqrt{2+\cos(t)}\,dt = \frac{d}{dx}\left[-\int_0^{x^2}\sqrt{2+\cos(t)}\,dt + \int_0^{x^3}\sqrt{2+\cos(t)}\,dt\right]$

$= -\sqrt{2+\cos(x^2)} \cdot 2x + \sqrt{2+\cos(x^3)} \cdot 3x^2$

chain Rule

(2) $f(x) = 1 + 2x + x^2 - x^3$

$f'(x) = 2 + 2x - 3x^2$

$f'(x)$ exists for all x, so no singular pts.

Endpoints: $\boxed{-\infty, \infty}$

Stationary Pts: Solve $f'(x) = 0$ for x:

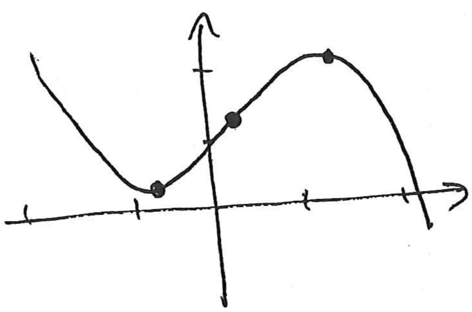
$$3x^2 - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 + 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \boxed{\frac{1 \pm \sqrt{7}}{3}} \approx -0.5486,\ 1.215$$

$f''(x) = 2 - 6x$

Inflection: Solve $f''(x) = 0$ for x:

$$2 - 6x = 0 \qquad x = \frac{1}{3}$$

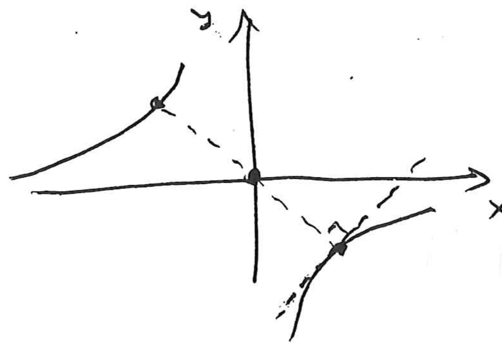


f is decreasing for $x < \frac{1-\sqrt{7}}{2}$

and for $x > \frac{1+\sqrt{7}}{2}$

f is concave up for $x < \frac{1}{3}$

(3)



constraint: $xy = -16$

Solve for y: $y = -\frac{16}{x}$

Distance^2 from (x,y) to $(0,0)$:

$$r^2 = x^2 + y^2 = x^2 + \frac{16^2}{x^2}$$

Stationary pts: Solve $\frac{d(r^2)}{dx} = 0$ for x:

$$\frac{d(r^2)}{dx} = 2x - 2\frac{16^2}{x^3} = 2\left(x - \frac{16^2}{x^3}\right) = \frac{2}{x^3}\left(x^4 - 16^2\right)$$

$$\frac{d(r^2)}{dx} = 0 \rightsquigarrow x^4 = 16^2, \quad x^2 = 16, \quad x = \pm 4$$

Ans: $(4,-4)$ & $(-4,4)$

Check: $y' = \frac{16}{x^2}$ $\quad y'(\pm 4) = 1 \leftarrow$ perp.!

(4)

$$w' = w^2(t+1)$$

$$\frac{dw}{dt} = w^2(t+1)$$

$$\int \frac{1}{w^2}\,dw = \int (t+1)\,dt$$

$\uparrow$ w^{-2}

$$\frac{w^{-2+1}}{-2+1} = \frac{t^2}{2} + t + C$$

$$\frac{w^{-1}}{-1} = \frac{t^2}{2} + t + C$$

$$w^{-1} = -\frac{t^2}{2} - t - C$$

$$w = \left[-\frac{t^2}{2} - t - C\right]^{-1}$$

$w(0) = 2$ so $\quad 2^{-1} = -C$

$\uparrow \frac{1}{2}$

Thus, $\boxed{w = \left[-\frac{t^2}{2} - t + \frac{1}{2}\right]^{-1} = \frac{1}{\frac{1}{2} - t - \frac{t^2}{2}}}$

$$= \frac{2}{1 - 2t - t^2}$$

<u>Check:</u> $w' = -\frac{1}{\left(\frac{1}{2} - t - \frac{t^2}{2}\right)^2} \cdot (-1-t)$

$$= \frac{t+1}{\left(\frac{1}{2} - t - \frac{t^2}{2}\right)^2} = w^2(t+1)$$

$w(0) = \frac{1}{\frac{1}{2}} = 2$