CAL I MAT1214.2
Fall 2001
Midterm 1

① a)

$$\begin{array}{r|l} & x^2-3x+9 \\ \hline x+3 & x^3+27 \\ & x^3+3x^2 \\ \hline & -3x^2+27 \\ & -3x^2-9x \\ \hline & 9x+27 \\ & 9x+27 \\ \hline & 0 \end{array}$$

$$\frac{x^3+27}{x^2-9} = \frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}(x-3)} \to \frac{(-3)^2-3(-3)+9}{-3-3}$$

$$= \frac{9+9+9}{-6} = \boxed{-\frac{9}{2}}$$

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b) If $x>-1$, then $|1+x| = 1+x$

$$\therefore \lim_{x\to -1^+} \frac{|1+x|}{1+x} = \lim_{x\to -1^+} \frac{1+x}{1+x} = \boxed{1}$$

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c) For t small $\tan(2t) \approx 2t$

$\cos(t^2) \approx 1$

$\sin(3t) \approx 3t$

So my guess is $\frac{2t \cdot 1}{3t} = \boxed{\frac{2}{3}}$

Verification: $$\frac{\tan(2t)\cdot\cos(t^2)}{\sin(3t)} = \underbrace{\frac{\sin(2t)}{2t}}_{\to 1} \cdot \frac{2t}{1} \cdot \underbrace{\frac{1}{\cos(2t)}}_{\to 1} \cdot \underbrace{\frac{3t}{\sin(3t)}}_{\to 1} \cdot \frac{1}{3t} \cdot \underbrace{\frac{\cos(t^2)}{1}}_{\to 1}$$

$$\to \frac{2t}{3t} = \boxed{\frac{2}{3}}$$

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d) $\cos(\frac{1}{t})$ is Bounded and $t \to 0$ so my guess is 0.

Verification: $-1 \le \cos(\frac{1}{t}) \le 1$

$$-|t| \le t\cos(\frac{1}{t}) \le |t|$$

$\to 0$ By Squeeze Law

(2) a) $f=\sqrt{x}=x^{\frac{1}{2}}$, $f'=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

b) $f=\frac{1}{\sqrt{3x^3}}=\frac{1}{\sqrt{3}}x^{-\frac{3}{2}}$, $f'=\frac{1}{\sqrt{3}}\cdot\left(-\frac{3}{2}\right)x^{-\frac{3}{2}-1}=-\frac{\sqrt{3}}{2}x^{-\frac{5}{2}}$

$=-\frac{\sqrt{3}}{2}\frac{1}{\sqrt{x^5}}$

c) $f=\frac{3x+1}{2x^3-5}$, $f'=\frac{3(2x^3-5)-(3x+1)6x^2}{(2x^3-5)^2}$

$=\frac{-3[4x^3+2x^2+5]}{(2x^3-5)^2}$

d) $f=2x^3\cos(x)$, $f'=2[3x^2\cos x+x^3(-\sin x)]$

$=2x^2[3\cos(x)-x\sin(x)]$

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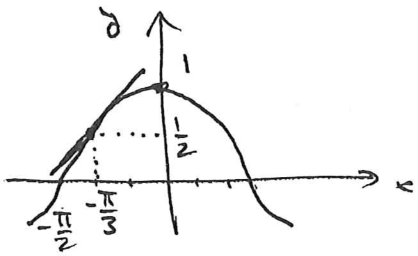
(3)



$y=\cos(x)$ $\quad y(-\frac{\pi}{3})=\frac{1}{2}$

$y'=-\sin(x)$

$m=y'(-\frac{\pi}{3})=-\sin(\frac{\pi}{3})=\frac{\sqrt{3}}{2}\approx 0.866$

Pt. Slope: $\boxed{y-\frac{1}{2}=\frac{\sqrt{3}}{2}(x+\frac{\pi}{3})}$

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(4) $f'(4)=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}$

$=\lim_{h\to 0}\frac{4+h-4}{h(\sqrt{4+h}+\sqrt{4})}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}=\boxed{\frac{1}{4}}$

Check: $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$, $f'(4)=\frac{1}{2}\cdot 4^{-\frac{1}{2}}=\frac{1}{2}\cdot\frac{1}{\sqrt{4}}=\frac{1}{4}$