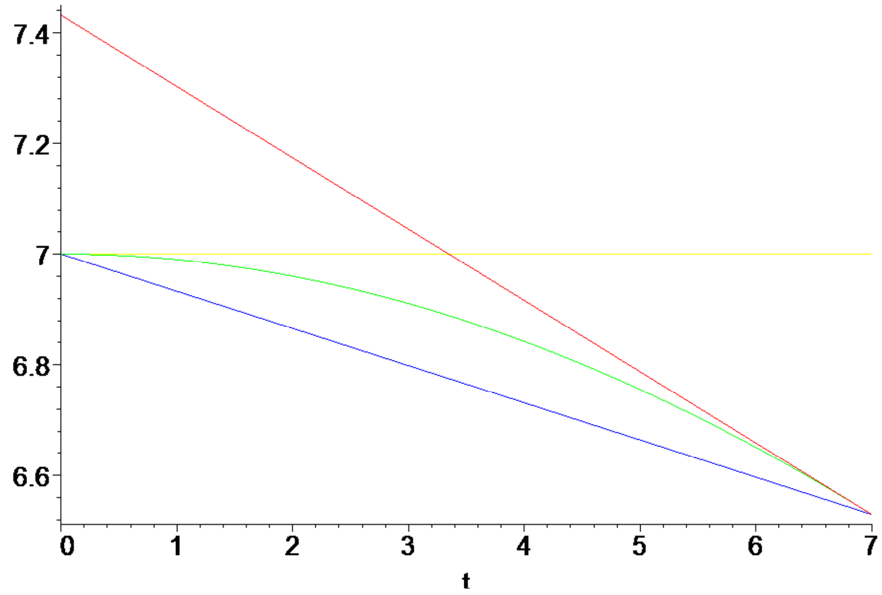
MAT1194.001 Fall '09 Midterm 1
#1
```
> p:=3*exp(k*t);
```
$$p := 3\,\mathbf{e}^{(k\,t)}$$
Use doubling time info to solve for k
```
> subs(t=5,p);
  kk:=solve(%=6,k);
  evalf(%);
```
$$3\,\mathbf{e}^{(5\,k)}$$
$$kk := \frac{1}{5}\ln(2)$$
$$0.1386294361$$
Plug k back in
```
> pp:=subs(k=kk,p);
  evalf(%);
```
$$pp := 3\,\mathbf{e}^{(1/5\,\ln(2)\,t)}$$
$$3.\,\mathbf{e}^{(0.1386294361\,t)}$$
Set the size to 80 and solve for t
```
> solve(pp=80,t);
  evalf(%);
```
$$\frac{5\ln\left(\frac{80}{3}\right)}{\ln(2)}$$
$$23.68482797$$
#2
```
> s:=5+2*cos(0.1*t);
```
$$s := 5 + 2\cos(0.1\,t)$$
```
> ds:=diff(s,t);
```
$$ds := -0.2\sin(0.1\,t)$$
Values of s at the endpoints (heights of the two points on the graph below)
```
> s0:=subs(t=0,s): simplify(%);
  s1:=subs(t=7,s): simplify(%);
```
$$7.$$
$$6.529684375$$
Average rate of change (slope of the secant line on the graph)
```
> ave:=(s1-s0)/7: evalf(%);
```
$$-0.0671879464$$
Instantaneous rates of change (derivative evaluated at the endpoints) (slope of the two tangent lines at the endpoints)
```
> inst0:=subs(t=0,ds): simplify(%);
  inst1:=subs(t=7,ds): simplify(%);
```
$$0.$$

$$-0.1288435374$$

Plot s(t) and the secant and tangent lines (use point-slope formula)

```
> plot({s,s0+ave*t,s0+inst0*t,s1+inst1*(t-7)},t=0..7);
```



#3

```
> 2^sin(t^3); diff(%,t);
```

$$2^{\sin(t^3)}$$

$$3\,2^{\sin(t^3)}\cos(t^3)\,t^2\ln(2)$$

```
> ln(t)/t; diff(%,t); simplify(%);
```

$$\frac{\ln(t)}{t}$$

$$\frac{1}{t^2}-\frac{\ln(t)}{t^2}$$

$$-\frac{-1+\ln(t)}{t^2}$$

#4

```
> f:=t/(1+t);
```

$$f := \frac{t}{1+t}$$

```
> diff(f,t); simplify(%);
  diff(%,t);
```

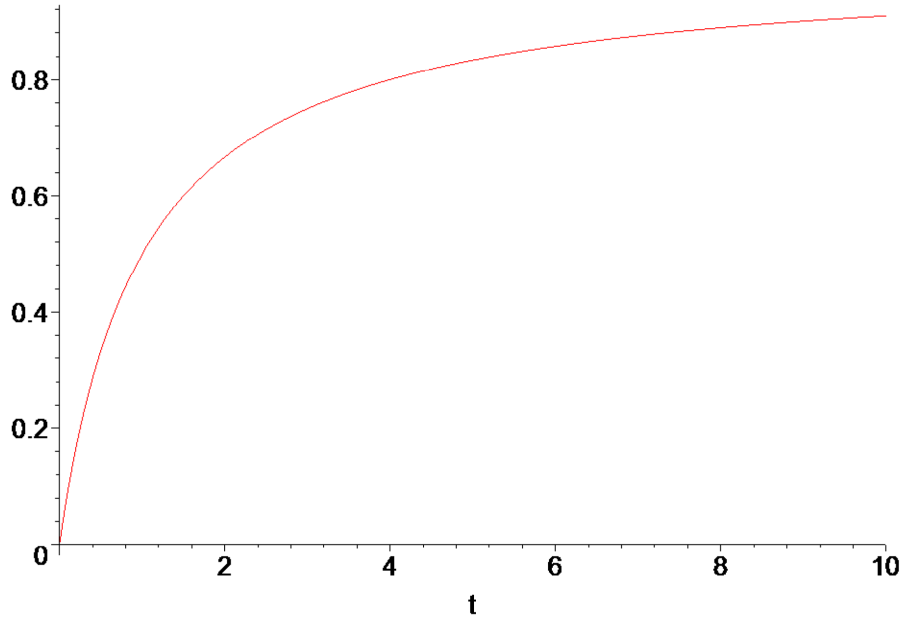
$$\frac{1}{1+t}-\frac{t}{(1+t)^2}$$

$$\frac{1}{(1+t)^2}$$

$$-\frac{2}{(1+t)^3}$$

For t geq 0 the second derivative is negative, so f(t) is concave down

```
> plot(f,t=0..10);
```



#5

```
> f:=x^2/(1+x^2);
```

$$f := \frac{x^2}{1+x^2}$$

Solve for fixed points of f (equilibria). Only one of the solutions (x=0) is real.

```
> x-f; simplify(%); solve(%,x);
```

$$x - \frac{x^2}{1+x^2}$$

$$\frac{x\,(1+x^2-x)}{1+x^2}$$

$$0, \frac{1}{2}+\frac{1}{2} I \sqrt{3}, \frac{1}{2}-\frac{1}{2} I \sqrt{3}$$

The slope of f at 0 is 0 which is less than 1 by magnitude. Therefore 0 is a stable equilibrium.

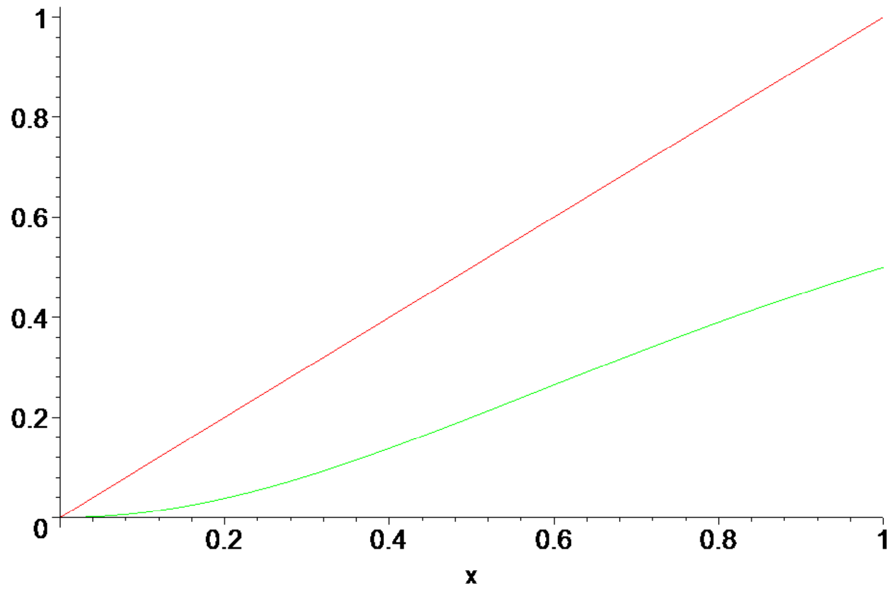
```
> diff(f,x); simplify(%);
  subs(x=0,%);
```

$$\frac{2\,x}{1+x^2} - \frac{2\,x^3}{(1+x^2)^2}$$

$$\frac{2\,x}{(1+x^2)^2}$$

$$0$$

```
> plot({f,x},x=0..1);
```


1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
x

>