(1) a) $\int \frac{t}{t^2+1}\,dt$

Let $u = t^2+1$. Then $\frac{du}{dt} = 2t$

$\therefore\ t\,dt = \frac{1}{2}\,du$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u = \boxed{\frac{1}{2}\ln(\underbrace{t^2+1}_{>0}) + C}$

b) $\int \frac{\ln t}{t}\,dt$ Let $u = \ln t$ $\frac{du}{dt} = \frac{1}{t}$

$\therefore\ \frac{1}{t}\,dt = du$

$= \int u\,du = \frac{1}{2}u^2 = \boxed{\frac{1}{2}(\ln t)^2 + C}$

c) $\int \underbrace{(t+1)}_{g}\ \underbrace{\cos(3t)}_{f'}\,dt$ $\int f'g = fg - \int fg'$

$g' = 1$ $f = \frac{1}{3}\sin(3t)$

$\frac{1}{3}(t+1)\sin(3t) - \frac{1}{3}\int \sin(3t)\,dt$

$\boxed{= \frac{1}{3}(t+1)\sin(3t) + \frac{1}{9}\cos(3t) + C}$

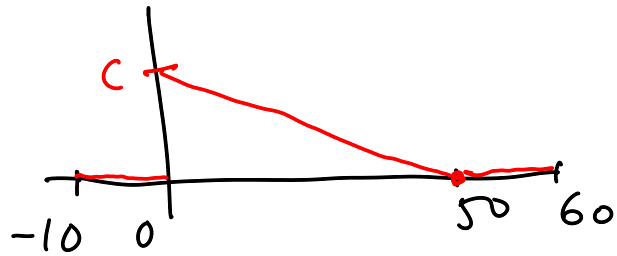
Shortcut:

$t+1$		$\cos(3t)$
	$\oplus$	
1	$\ominus$	$\frac{1}{3}\sin(3t)$
0		$-\frac{1}{9}\cos(3t)$

②

$$p(t) = \begin{cases} c\,(1 - 0.02\,t) & \text{for } 0 \le t \le 50 \\ 0 & \text{otherwise} \end{cases} \quad (c>0)$$

$p(0) = c \qquad p(50) = 0$



$p(t) \geq 0$ ☺

Require $1 = \int_{-\infty}^{\infty} p(t)\,dt = \int_0^{50} c\,(1 - 0.02t)\,dt$

$= c\left[t - 0.01t^2\right]_0^{50} = c\,(50 - 0.01 \cdot 50^2)$

$= 50c\,(1 - 0.01 \cdot 50) = 50c\,(1 - 0.5)$

$= 25c$

$\therefore \boxed{c = \frac{1}{25} = 0.04}$

$\therefore p(t) = 0.04(1 - 0.02t)$

cdf: $P(t) = 0.04\,(t - 0.01t^2)$

b) $\int_{30}^{50} p(t) = P(t)\Big|_{30}^{50}$

$= 0.04\left[50 - 0.01 \cdot 50^2 - (30 - 0.01 \cdot 30^2)\right]$

$= \boxed{0.16 = 16\%}$

c) Mean: $\int_{-\infty}^{\infty} t\, p(t)\, dt = \int_0^{50} 0.04\, t\, (1 - 0.02t)\, dt$

$$= 0.04 \int_0^{50} (t - 0.02t^2)\, dt = 0.04 \left[\frac{1}{2} t^2 - \frac{0.02}{3} t^3 \right]_0^{50}$$

$$= 0.04 \left[\frac{1}{2} 50^2 - \frac{0.02}{3} 50^3 \right] = \boxed{16.7 \text{ sec}}$$

d) Median: Require $P(t) = \frac{1}{2}$ (solve for t)

$$0.04\,(t - 0.01t^2) = \frac{1}{2}$$

$$t - 0.01t^2 = \frac{0.5}{0.04} = 12.5$$

$$0.01t^2 - t + 12.5 = 0$$

$$t^2 - 100t + 1250 = 0$$

$x^2 + 2px + q = 0$

$x = -p \pm \sqrt{p^2 - q}$

$$t = 50 \pm \sqrt{50^2 - 1250} = 50 \pm \sqrt{1250}$$

$$= 50 \pm 35.355$$

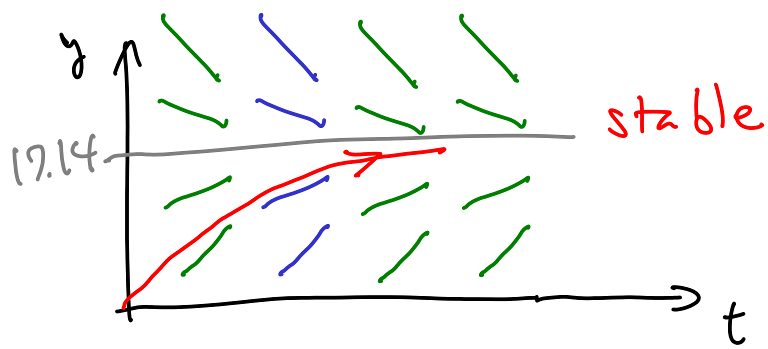
$50 + \sqrt{1250}$ is outside the interval $0 \le t \le 50$ ☹

Median: $50 - 35{,}355 = \boxed{14.6 \text{ sec}}$

③ a) $\boxed{\dfrac{dy}{dt} = 6 - 0.35\,y}$ Autonomous ☺

b) Equi: $\dfrac{dy}{dt} = 0 \Rightarrow 6 - 0.35\,y = 0$

$$y = \frac{6}{0.35} = \boxed{17.14 \text{ mg}}$$

c) 

d) $\dfrac{dy}{dt} = 6 - 0.35\,y$

$$= -0.35(y - 17.14)$$

$$y = 17.14 + C e^{-0.35t}$$

$\dfrac{dy}{dt} = k(y - A)$

$y = A + Ce^{kt}$

$y(0) = 0 \Rightarrow \boxed{y = 17.14 - 17.14\, e^{-0.35t}}$

④ $\dfrac{dy}{dt} = \dfrac{y}{t+1}$ $\qquad y(0) = 5$

a) $\displaystyle\int \frac{dy}{y} = \int \frac{dt}{t+1}$

$\| \qquad\qquad \|$

$\ln \underbrace{y}_{>0} \qquad \ln(\underbrace{t+1}_{\substack{>0 \\ \text{for } t \geq 0}}) + C$

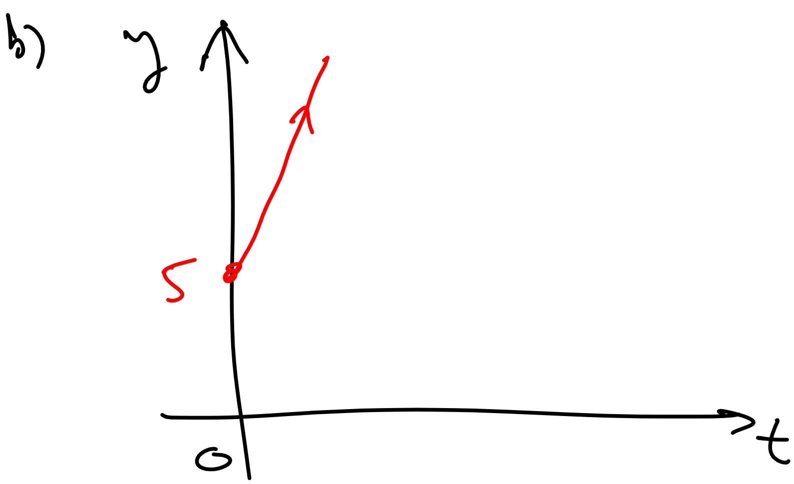
$y = e^{\ln y} = e^{\ln(t+1)+C} = e^{\ln(t+1)} \cdot e^{C}$

$= (t+1)e^{C}$

Plug in $t=0$, $y=5$: $\quad 5 = e^{C}$

$\therefore \quad y = (t+1)\cdot 5 = \boxed{5t+5}$

b)



As t grows to ∞
$y(t)$ grows linearly without bound

$\left(\lim_{t \to \infty} y(t) = \infty \right)$