1. Exponential model $b(t) = b(0)a^t$ $(a>0)$

Given: $b(1) = 30$ $\qquad b(0)a = 30$

$b(2) = 45$ $\qquad b(0)a^2 = 45$ $\Big\}$ divide

$a = \frac{3}{2}$

$b(0) \cdot \frac{3}{2} = 30$ $\quad \therefore b(0) = 20$

$$b(7) = b(0)a^7 = 20\left(\frac{3}{2}\right)^7 = \frac{10935}{3} \approx 341.7$$

$\therefore$ In a week after the start Perry has 341.7 million bacteria.

(2) Let $p(t) = \frac{1000}{t^2} = 1000\, t^{-2}$

Then $p'(t) = 1000(-2)t^{-2-1} = -\frac{2000}{t^3}$

From definition $p'(t) = \lim_{h \to 0} \frac{p(t+h) - p(t)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1000}{(t+h)^2} - \frac{1000}{t^2}}{h} = \lim_{h \to 0} 1000 \frac{t^2 - (t+h)^2}{(t+h)^2 t^2 h}$$

$$= \lim_{h \to 0} 1000 \frac{\cancel{t^2} - (\cancel{t^2} + 2th + h^2)}{(t+h)^2 t^2 h} = \lim_{h \to 0} 1000 \frac{-2t\cancel{h} - h^{\cancel{2}}}{(t+h)^2 t^2 \cancel{h}}$$

$$= \lim_{h \to 0} -1000 \frac{2t + h}{(t+h)^2 t^2} = \frac{-2000\cancel{t}}{t^2 \cdot t^{\cancel{2}}} = -\frac{2000}{t^3}$$

a) $p'(21) = -\frac{2000}{9261} \approx -0.216$

$\therefore$ When Tucker is 21, his credibility is dropping by <u>0,216</u> pts per year

b) $\frac{p(25) - p(21)}{25 - 21} = \frac{8/5 - \frac{1000}{41}}{4} = -\frac{368}{2205} \approx -0,167$

$\therefore$ On average Tucker's popularity between ages 21 & 25 was dropping by <u>0.167</u> pts/y

3) a) $$(t^4 a^{b^t})' = (t^4)' a^{b^t} + t^4 (a^{b^t})'$$

$(a^x)' = \ln a \, a^x$

$$= 4t^3 a^{b^t} + t^4 \ln a \; a^{b^t} (b^t)'$$

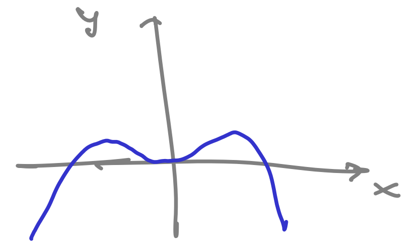
$$= \underline{4t^3 a^{b^t} + t^4 \ln a \; a^{b^t} \ln b \; b^t}$$

$$= a^{b^t} \left(4t^3 + \ln a \ln b \; t^4 b^t \right)$$

b) $$\left(\frac{\ln t}{t} \right)' = \frac{(\ln t)' t - \ln t \cdot t'}{t^2}$$

$$= \frac{\frac{1}{\cancel{t}} \cdot \cancel{t} - \ln t \cdot 1}{t^2} = \boxed{\frac{1 - \ln t}{t^2}}$$

(4) $f(x) = 2x^2 - x^4$



a) $f'(x) = 4x - 4x^3$

$= 4x(1-x^2) = 4x(1-x)(1+x)$

$f'(x)$ exists for all x

$f'(x) = 0 \Rightarrow x = 0, \pm 1$

$\therefore$ Critical pts of f are $x = 0, 1, -1$

$f''(x) = 4 - 12x^2$

$f(0) = 4 > 0$
$f(1) = -8 < 0$
$f(-1) = -8 < 0$

$\therefore$ $[0, 0]$ is a local min

$\left.\begin{array}{l}[1, 1] \\ [-1, 1]\end{array}\right\}$ local maxima (global max = 1)

b) $f''(x)$ exists for all x

$f''(x) = 0 \Rightarrow 4 - 12x^2 = 0$

$1 - 3x^2 = 0$

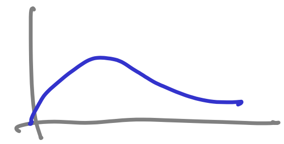
$x^2 = \frac{1}{3}$ $\quad x = \pm \frac{1}{\sqrt{3}} \approx \pm 0.577$

If $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, $x^2 < \frac{1}{3}$, $f''(x) > 0$, so f is concave up

If $x < -\frac{1}{\sqrt{3}}$ or $x > \frac{1}{\sqrt{3}}$, $x^2 > \frac{1}{3}$, $f''(x) < 0$, so f is concave down

$\therefore$ $\left[\frac{1}{\sqrt{3}}, \frac{5}{9}\right]$, $\left[-\frac{1}{\sqrt{3}}, \frac{5}{9}\right]$ are inflection pts

(5) Let $b(t) = 3te^{-0.2t}$



$$b'(t) = 3\left[t'e^{-0.2t} + t\left(e^{-0.2t}\right)'\right]$$
$$= 3\left[1\cdot e^{-0.2t} + te^{-0.2t}(-0.2)\right]$$
$$= 3e^{-0.2t}\left[1 - 0.2t\right]$$

a) $b'(0) = 3$ $\therefore$ initially concentration rises at 3 mg/cc/h

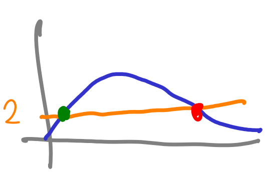
b) $b'(t)$ exists for all t

$b'(t) = 0 \Rightarrow 1 - 0.2t = 0 \Rightarrow t = 5$

	t	$b(t)$	
crit. pt. →	5	5.518	← global max
End pt. →	0	0	
	∞	$\lim\limits_{t\to\infty} 3t\underbrace{e^{-0.2t}}_{\text{dominant}} = 0$	

$\therefore$ maximum concentration is 5.518 mg/cc and it occurs 5 hrs after the injection

c) Solve for t $b(t) = 2$:



$3te^{-0.2t} = 2$ numerically solve

$t \approx \underline{0.779}, \underline{15.84}$

$\therefore$ the concentration starts being effective after 0.779 hrs and stops being effective after $\underline{15.84h}$