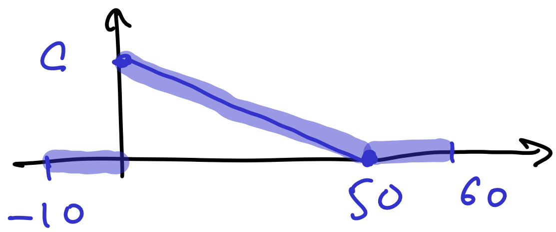
(1) $$p(t) = \begin{cases} c(1-0.02t) & 0 \le t \le 50 \\ 0 & \text{otherwise} \end{cases}$$

a)



$p(0) = c$

$p(50) = 0$

$$1 = A = \frac{1}{2} c \cdot 50 = 25c$$

$$\therefore \quad c = \frac{1}{25} = 0.04$$

b) $$\text{Prob}(t \le 10) = \int_0^{10} p(t)\,dt$$

$$= \int_0^{10} c(1-0.02t)\,dt = \underbrace{c\left[t - 0.02\frac{t^2}{2}\right]}_{cdf}\Big|_0^{10}$$

0.01

Check: $cdf(50) = 0.04\left[50 - 0.01 \cdot 50^2\right] =$

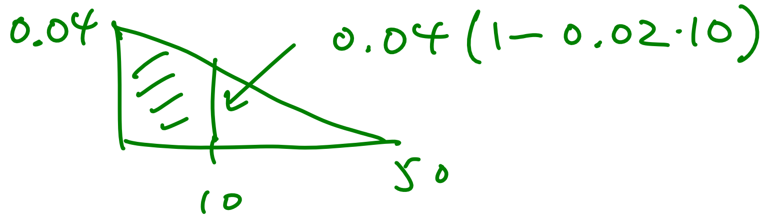
$= 0.04 \cdot 50\left[1 - 0.01 \cdot 50\right] = 2\left[1 - 0.5\right]$

$= 2 \cdot 0.5 = 1$

$cdf(10) = 0.04 \cdot 10\left[1 - 0.01 \cdot 10\right]$

$= 0.4 \cdot 0.9 = \boxed{0.36}$

Check:



$$10 \cdot \frac{0.04 + 0.04(1-0.2)}{2} = 0.02\,(1+1-0.2)$$

$$= 0.02 \cdot 1.8 = .36$$ ☺

∴ 36% of tattoos heal by the first 10 days.

c) $$\mu = \int_{-\infty}^{\infty} t\, p(t)\, dt = \int_0^{50} t \cdot c(1-0.02t)\, dt$$

$$= 0.04 \int_0^{50} (t - 0.02t^2)\, dt$$

$$= 0.04 \left[\frac{t^2}{2} - 0.02\frac{t^3}{3} \right]_0^{50}$$

$$= 0.04 \left(\frac{50^2}{2} - 0.02\frac{50^3}{3} \right) = \boxed{16.666\ldots}$$

∴ On average a tattoo takes 16.7 days to heal.

d) Set $cdf(t) = \frac{1}{2}$, solve for t:

$$0.04\left(t - 0.01\, t^2\right) = 0.5$$

$$t - 0.01\, t^2 = \frac{0.5}{0.04} = 12.5$$

$$0.01\, t^2 - t + 12.5 = 0$$

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{1 \pm \sqrt{1 - 4 \cdot 0.01 \cdot 12.5}}{0.02} = \cancel{85.4},\ \boxed{14.64466}$$

$\therefore$ By 14.6 days half the tattoos are healed.

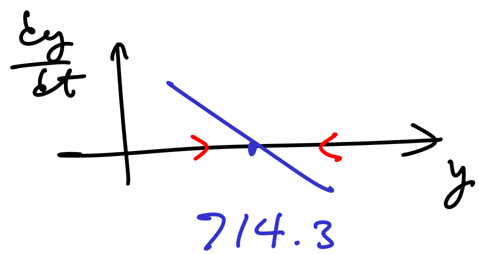
(2)

a) $\boxed{\dfrac{dy}{dt} = 500 - 0.7y}$

b) Equi $\Rightarrow$ $\dfrac{dy}{dt} = 0$ $\Rightarrow$ $y = \dfrac{500}{0.7} = \boxed{714.3}$

$\therefore$ One equi.: 714.3 mg

c)



$\therefore$ 714.3 is a stable equi.

d) In the long run the amount of drug stabilizes at 714.3 mg

(3) $\dfrac{dB}{dt} = \dfrac{2B}{t}$ $\qquad B(0) = 5$

$\displaystyle\int \frac{dB}{B} = 2\int \frac{dt}{t}$, $\quad \ln|B| = 2\ln|t| + C$

Since $B, t > 0$ $\qquad \ln B = 2\ln t + C$

$B = e^{2\ln t + C} = e^{C} \cdot \left(e^{\ln t}\right)^2 = e^{C} t^2$

Gen. sol: $\boxed{B(t) = kt^2}$

$B(0) = 0 \neq 5$ No particular solution.