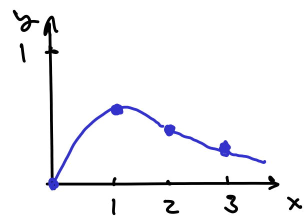
# MAT 1193.002 Midterm 2 Fall 2014

1) a)

| $x$ | $0$ | $1$ | $2$ | $3$ | $\to\infty$ |
| --- | --- | --- | --- | --- | --- |
| $y$ | $0$ | $\frac{1}{2}$ | $\frac{2}{5}$ | $\frac{3}{10}$ | $\to 0$ |




$$y = \frac{x}{1+x^2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

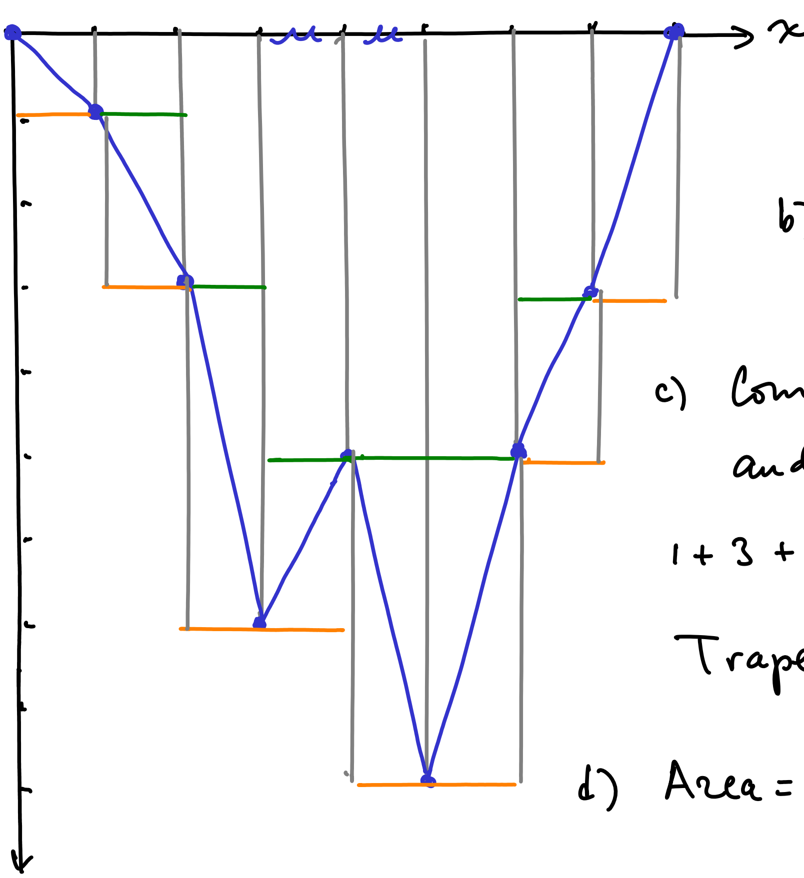
$$\frac{dy}{dx} = \frac{x'(1+x^2) - x(1+x^2)'}{(1+x^2)^2} = \frac{1+x^2 - x\cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$\therefore$ For $x \geq 0$ $\max y = \frac{1}{2}$ at $\boxed{x = 1}$

b) The maximum fraction cancer free is half. It is achieved with dosage 1 mg

2) a)




b) Lower estimate (green)

$1 + 3 + 5 + 5 + 5 + 3 = \boxed{22 \text{ ft}^2}$

c) Compute upper estimate (orange) and average (trapezoidal)

$1 + 3 + 7 + 7 + 9 + 9 + 5 + 3 = 44 \text{ ft}^2$

$$\text{Trapezoidal} = \frac{44 + 22}{2} = \boxed{33 \text{ ft}^2}$$

d) $\text{Area} = \int_0^8 y(x)\,dx$

③ a) $\int \underbrace{(\cos t)^2}_{u^2} \underbrace{\sin t \, dt}_{-du}$

Let $u = \cos t$, then $du = -\sin t \, dt$

$$= -\int u^2 du = -\frac{1}{3}u^3 = \boxed{-\frac{1}{3}(\cos t)^3 + C}$$

b) $\int \frac{4^{\sqrt{t}}}{\sqrt{t}} dt$ $\leftarrow 4^u$, $2du$

Let $u = \sqrt{t}$, then $du = \frac{1}{2\sqrt{t}} dt$

$\left[ (t^{1/2})' = \frac{1}{2} t^{-\frac{1}{2}} \right]$

$$= 2\int 4^u du = 2 \cdot \frac{1}{\ln 4} 4^u = \boxed{\frac{1}{\ln 2} 4^{\sqrt{t}} + C}$$

c) $\int \underbrace{t}_{v} \underbrace{2^{3t+1}}_{u'} dt$

$\int u'v = uv - \int uv'$

$v' = 1$, $u = \int u' = \int 2^{3t+1} dt = $ $\leftarrow 2^w$, $\leftarrow \frac{1}{3}dw$

Let $w = 3t+1$ then, $dw = 3dt$

$= \frac{1}{3}\int 2^w dw = \frac{1}{3\ln 2} 2^w = \frac{1}{3\ln 2} 2^{3t+1}$

$$= \frac{t}{3\ln 2} 2^{3t+1} - \frac{1}{3\ln 2}\int 2^{3t+1} dt$$

$$\boxed{= \frac{t}{3\ln 2} 2^{3t+1} - \frac{1}{(3\ln 2)^2} 2^{3t+1} + C} = \frac{2^{3t+1}}{3\ln 2}\left[t - \frac{1}{3\ln 2}\right] + C$$

**Shortcut:**

differentiate ↓ ; integrate ↓

| | |
|---|---|
| $t$ | $2^{3t+1}$ |
| $1$ | $+$ $\frac{1}{3\ln 2} 2^{3t+1}$ |
| $0$ | $-$ $\frac{1}{(3\ln 2)^2} 2^{3t+1}$ |