MAT 1193.002 Midterm 1 Fall 2014

① Let $b(t)$ be the size of the colony (in millions) at time t (days).

$b(2)=7$, $b(2+3)=b(5)=8$. $b(5+4)=b(9)=?$

Exponential model: $b(t)=b(0)\,a^t$ for some $a>0$.
(rate: $r=\ln a$)

$7=b(2)=b(0)\,a^2$

$8=b(5)=b(0)\,a^5$

divide: $\frac{7}{8}=\frac{\cancel{b(0)}\,a^2}{\cancel{b(0)}\,a^5}$

$\frac{a^5}{a^2}=a^{5-2}=a^3=\frac{8}{7}$ $\therefore$ $a=\sqrt[3]{\frac{8}{7}}\approx \underline{1.0455159}$

$b(0)=7\cdot a^{-2}\approx \underline{6.40}$

$b(9)=b(0)\,a^9\approx \underline{9.55900}$

[$a=2\cdot 7^{-1/3}$, $b(0)=2^{-2}\cdot 7^{5/3}$, $b(9)=2^7\cdot 7^{-4/3}$]

$\therefore$ After 9 days Justin Bieber will have 9.559 million bacteria.

② a) Let $\Delta t=10^{-3}$ $y'(5)\approx\frac{\Delta y}{\Delta t}=\frac{y(5+\Delta t)-y(5)}{\Delta t}=$

$=[y(5.001)-y(5)]10^3=6\cdot 10^3[2^{5.001}-2^5]\approx 133.130$

With $\Delta t=-10^{-3}$ $y'(5)\approx -6\cdot 10^3[2^{4.999}-2^5]\approx 133.038$

$\therefore$ $y'(5)$ is between 133.038 and 133.130

[$y'(5)=6\cdot\ln 2\cdot 2^5\approx 133.084$]

b) DNA grows at the rate of approximately 133 ng/min.

③

t	0	1	2.1	3.2	5
slope	2.4	0	-1	0	3.5

t	$0 \le t < 2$	$2 < t \le 5$
slope	$-3/2$	$5/3$