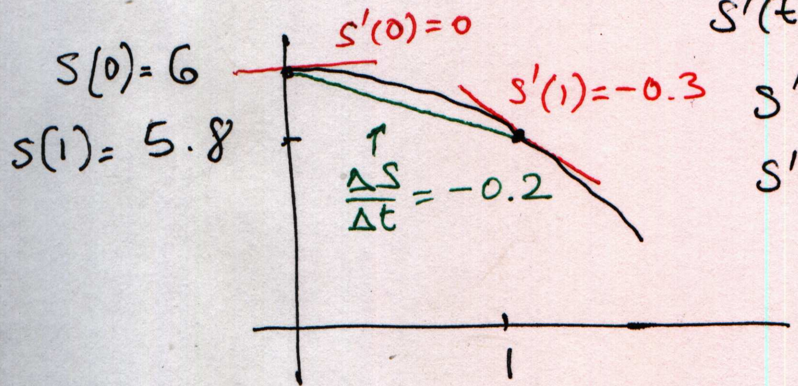
① $p_t = p_0 a^t$ ← exponential model

$$\left.\begin{aligned} 40 &= p_0 a \\ 60 &= p_0 a^2 \end{aligned}\right\} \text{divide:} \quad a = 1.5$$

$$40 = p_0 \cdot 1.5 \quad \Rightarrow \quad p_0 \approx 26.7$$

$\therefore$ Initial size is 26.7 million bacteria.

② $s(t) = 4 + 2\cos(0.4t)$



$s'(t) = -0.8\sin(0.4t)$

$s'(0) = 0$

$s'(1) = -0.3$

$$\frac{\Delta S}{\Delta t} = \frac{5.8-6}{1} = -0.2$$

③ a) $(t\,3^{2^t})' = t'\,3^{2^t} + t\left(3^{2^t}\right)$

$$= 3^{2^t} + \ln 3 \cdot 3^{2^t} \cdot \ln 2 \cdot 2^t$$

$$= 3^{2^t}\left(1 + \ln 3 \cdot \ln 2 \cdot 2^t\right)$$

b) $$\left(\frac{\ln t}{\sqrt{t}}\right)' = \frac{(\ln t)'\sqrt{t} - \ln t\,(\sqrt{t})'}{t} = \frac{\frac{1}{t}\sqrt{t} - \ln t\,\frac{1}{2\sqrt{t}}}{t}$$

$$= \frac{1 - \frac{1}{2}\ln t}{t\sqrt{t}}$$

4 $f(t) = \dfrac{1}{t^2+1}$, $f'(t) = \dfrac{-2t}{(t^2+1)^2}$

$$f''(t) = -2\,\frac{t'(t^2+1)^2 - t\left((t^2+1)^2\right)'}{(t^2+1)^4}$$

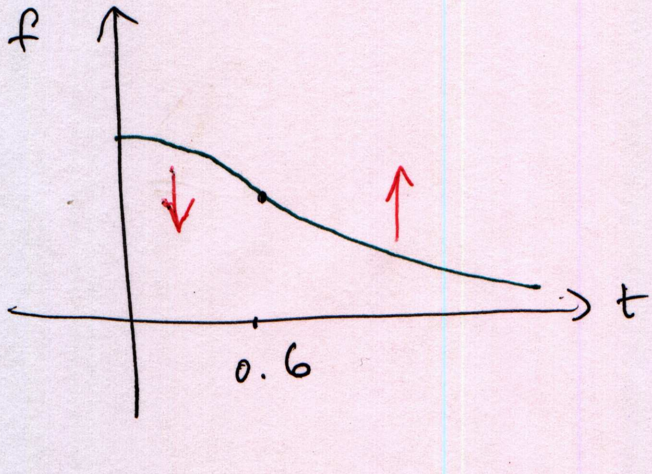
$$= -2\,\frac{(t^2+1)^{\cancel{2}} - t\cdot 2\cancel{(t^2+1)}\cdot 2t}{(t^2+1)^{\cancel{4}\,3}}$$

$$= -2\,\frac{t^2+1-4t^2}{(t^2+1)^3} = 2\cdot\frac{3t^2-1}{(t^2+1)^3}$$

$f''=0$ when $t = \pm\frac{1}{\sqrt{3}}$

$f''<0$ so f is concave down for $t<\frac{1}{\sqrt{3}}\approx 0.6$

$f''>0$ so f is concave up for $t>\frac{1}{\sqrt{3}}\approx 0.6$

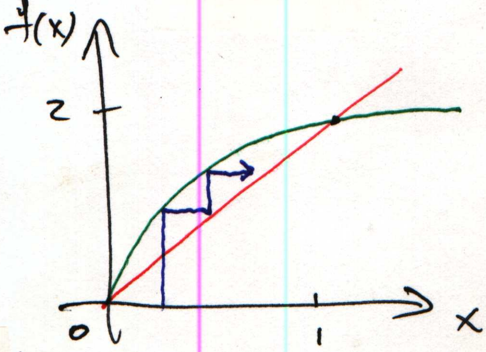


(5) $x_{t+1} = f(x_t)$ where $f(x) = \frac{2x}{x+1}$

$x = f(x) \Rightarrow x = \frac{2x}{x+1} \Rightarrow x = 0$ or $1 = \frac{2}{x+1}$

so $x = 1$

Cobwebbing shows that $x=0$ is an unstable equilibrium and $x=1$ is stable



$$f'(x) = 2\frac{x'(x+1) - x(x+1)'}{(x+1)^2} = 2\frac{\cancel{x}+1-\cancel{x}}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$f'(0) = 2$, $f'(1) = \frac{1}{2}$

In the long run the population tends to 1.