Midterm 1 / 2001.10.17 / Precalculus (Honors) / MAT 1093.002

Name: _____

Please show all work and provide clear explanations. Sketch. Make conclusions.

1. (24 pts.) Let ℓ_1 be the line in the plane passing through the points $(-2, 0)$ and $(0, -3)$. Let ℓ_2 be the line through the origin that is perpendicular to ℓ_1.

   (a) Sketch both lines on a properly labelled graph.

   (b) Find equations for the lines ℓ_1 and ℓ_2.

   (c) Find the point of intersection of ℓ_1 and ℓ_2.

2. (30 pts.) For each of the following functions, find the domain and range; determine whether the function has an inverse; if so, find the inverse using the same representation technique (formula, table, graph) as for the given function.

   (a) $f(x) = \dfrac{1}{x^2 + 1}$

   (b) $f(x) = (x - 2)^{-\frac{1}{3}}$

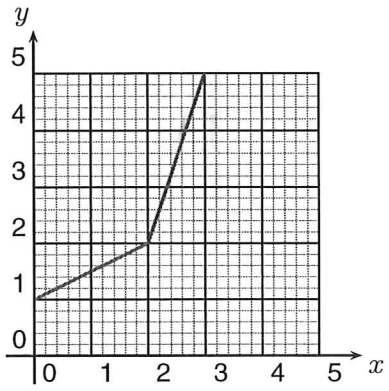
   (c)

x	0	0.1	0.2
$f(x)$	1.2	1.1	1.3

   (d) $f(x) = 1 - |x|$

   (e) $f(x) = \begin{cases} x - 1 & \text{for } x < 0 \\ x + 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \end{cases}$

   (f)



3. (24 pts.) One Friday, on the way home from work, Homer Simpson accidentally drops a chunk of radioactive material in the gents in Moe's bar. Moe discovers the chunk on Monday and estimates its weight to be 123.4 grams. The following Monday the chunk weighs 120.7 grams.

   (a) What is the daily rate of radioactive decay of this particular compound?

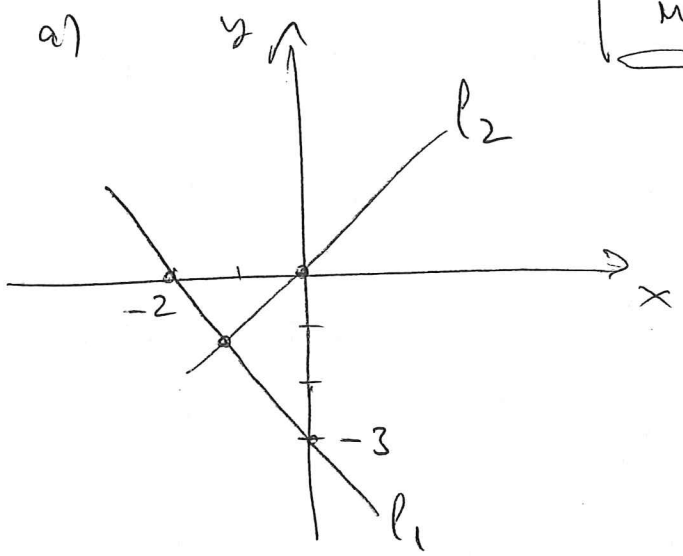
   (b) How much did the chunk weigh when Homer dropped it?

   (c) What is the compound's half life?

1	2	3	total (78)	(%)

MAT 1093 Precalculus
Midterm 1 Fall 2001

(1) a)



b) l_1: $y = -\frac{3}{2}x - 3$ $\quad$ l_2: $y = \frac{2}{3}x$

c) Solve $\frac{2}{3}x = -\frac{3}{2}x - 3$ for x:

$$\left(\frac{3}{2} + \frac{2}{3}\right)x = -3$$

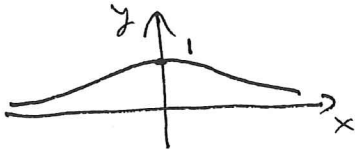
$$\frac{13}{6}x = -3$$

$$x = -\frac{18}{13}$$

Plug-in: $y = \frac{2}{3}\left(-\frac{18}{13}\right) = -\frac{12}{13}$

Pt. $\cap$ intersection: $\left(-\frac{18}{13}, -\frac{12}{13}\right)$

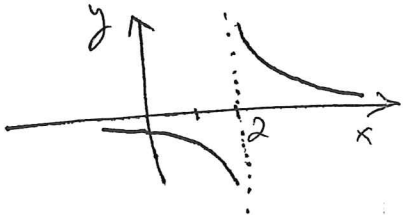
(2)

a) 

Domain: $\mathbb{R}$

Range: $(0,1] = \{y : 0 < y \leq 1\}$

Inverse function: does not exist

b) 

Domain: $\mathbb{R} \setminus \{2\} = \{x : x \neq 2\}$

Range: $\mathbb{R} \setminus \{0\} = \{y : y \neq 0\}$

Inverse: Solve $y = (x-2)^{-\frac{1}{3}}$ for x

$y^{-3} = x - 2$ $\qquad x = y^{-3} + 2$

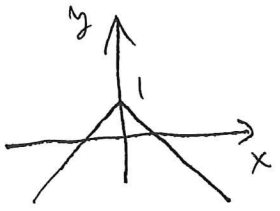
Inverse: $f^{-1}(y) = \frac{1}{y^3} + 2$

c) Domain: $\{0, 0.1, 0.2\}$

Range: $\{1.2, 1.1, 1.3\}$

Inverse:

y	1.1	1.2	1.3
$f^{-1}(y)$	0.1	0	0.2

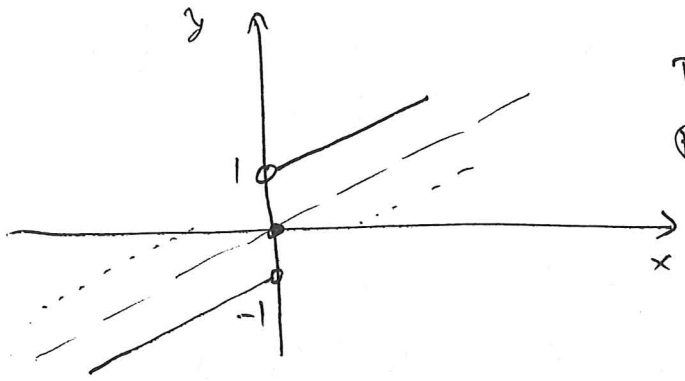
d) 

Domain: $\mathbb{R}$

Range: $(-\infty, 1] = \{y : y \leq 1\}$

No inverse

c)

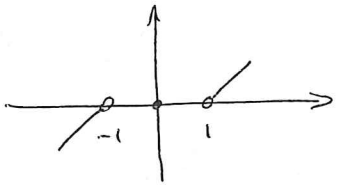


Domain: $\mathbb{R}$

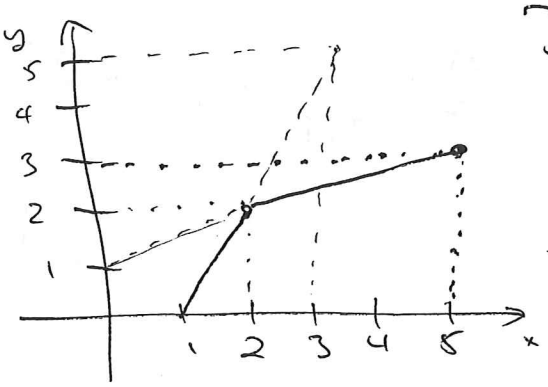
Range: $(-\infty,-1) \cup \{0\} \cup (1,\infty)$

$= \{y : y>1 \text{ or } y=0 \text{ or } y<-1\}$

Inverse: $f^{-1}(y) = \begin{cases} y+1 & \text{for } y<-1 \\ 0 & \text{for } y=0 \\ y-1 & \text{for } y>1 \end{cases}$



d)



Domain: $[0,3]$

Range: $[1,5]$

Inverse:

(3) Let t = time since Moe discovers the chunk (in days)

$A(t)$ = amount of material (in grams) at time t.

Exponential model: $A(t) = a\,b^t$

where $a = A(0) = 123.4$ g

$A(7) = 120.7$ g

So $123.4\ b^7 = 120.7$

So $b^7 = \frac{120.7}{123.4}$

So $b = \sqrt[7]{\frac{120.7}{123.4}} = \left(\frac{120.7}{123.4}\right)^{1/7} = 0.997$

So $A(t) = 123.4\,[0.997]^t$

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a) $b = 1 + r$, so $r = b - 1 = -0.003 = -0.3\ \%$

b) $A(-3) = 123.4\,[0.997]^{-3} = 124.5$ g

c) Solve $A(t) = \frac{1}{2} A(0)$ for t.

~~$123.4\,[0.997]^t = \frac{1}{2}\,123.4$~~

$[0.997]^t = \frac{1}{2}$

take ln of both sides $\Rightarrow$ $t \ln[0.997] = \ln\frac{1}{2} = -\ln 2$

$t = -\frac{\ln 2}{\ln[0.997]} = 231$ (approx) days