

①

$$x_n = x_{n-1} + 6x_{n-2}$$

$$x_0 = 2, x_1 = 1$$

$$x_2 = x_1 + 6x_0 = 13$$

$$x_3 = x_2 + 6x_1 = 19$$

Plug in r^n

$$r^n = r^{n-1} + 6r^{n-2}$$

$$r^2 - r - 6 = 0$$

$$r = 3, -2$$

$$r = A 3^n + B(-2)^n$$

$$A + B = 2$$

$$3A - 2B = 1$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -5 & -5 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore A = B = 1$$

$$x_n = 3^n + (-2)^n$$

$$n=0 \quad 2 \quad \checkmark$$

$$n=1 \quad 3-2 = 1 \quad \checkmark$$

$$n=2 \quad 9+4 = 13 \quad \checkmark$$

$$n=3 \quad 27-8 = 19 \quad \checkmark$$

☺

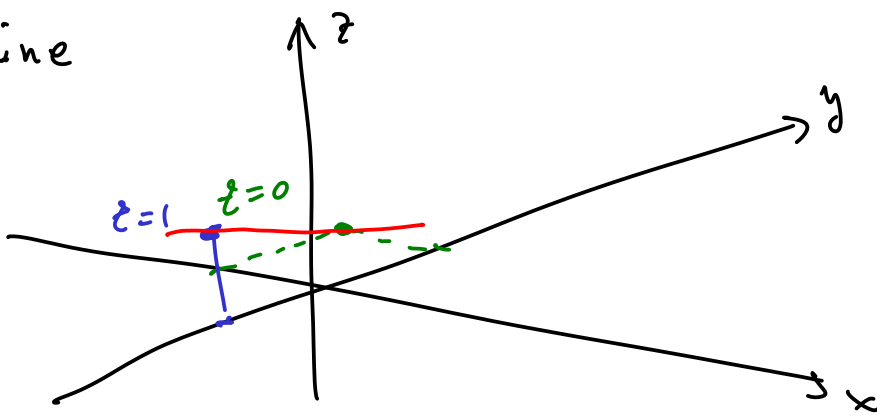
(2) $x = 1 - 2y - 3z$, $5y = 1 - 4x - 6z$, $9z = 1 - 7x - 8y$

a) $A = \begin{bmatrix} 1 & 2 & 3 & \vdots & 1 \\ 4 & 5 & 6 & \vdots & 1 \\ 7 & 8 & 9 & \vdots & 1 \end{bmatrix}$ ^{b)} row reduction $\rightarrow \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

↑
free

c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z - 1 \\ 1 - 2z \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Line



$z=1$
 $\Rightarrow \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$$\textcircled{3} \quad y = -3x$$

a) Projection $\text{proj}_{\hat{u}} \bar{x} = (\hat{u} \cdot \bar{x}) \hat{u}$

let $\bar{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ $|\bar{u}| = \sqrt{10}$ $\hat{u} = \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix}$

$$\text{proj}_{\hat{u}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} \frac{1}{\sqrt{10}} \\ -3/\sqrt{10} \end{bmatrix} = \begin{bmatrix} 1/10 \\ -3/10 \end{bmatrix}$$

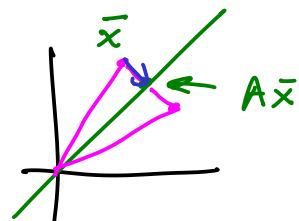
$$\text{proj}_{\hat{u}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\frac{3}{\sqrt{10}} \begin{bmatrix} 1/\sqrt{10} \\ -3/\sqrt{10} \end{bmatrix} = \begin{bmatrix} -3/10 \\ 9/10 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/10 & -3/10 \\ -3/10 & 9/10 \end{bmatrix}$$

$$\begin{bmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{bmatrix}$$

b) $B = 2A - I$

$$= \begin{bmatrix} -4/5 & -3/5 \\ -3/5 & 4/5 \end{bmatrix}$$



$$A\bar{x} - \bar{x}$$

$$\bar{x} + 2(A\bar{x} - \bar{x})$$

$$= 2A\bar{x} - \bar{x}$$

$$= (2A - I)\bar{x}$$

c) $A^2 = A$

$$B^2 = I$$

$$\text{proj}(\text{proj}) = \text{proj}$$

$$\text{refl.}^2 = \text{id}$$

$$\textcircled{4} \text{ a) } A = \begin{pmatrix} -7 & 3 \\ -18 & 8 \end{pmatrix}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} -7-\lambda & 3 \\ -18 & 8-\lambda \end{pmatrix}$$

$$= (-7-\lambda)(8-\lambda) + 54 = \lambda^2 - 8\lambda + 7\lambda - 56 + 54$$

$$= \lambda^2 - \lambda - 2$$

$$\lambda = 2, -1$$

$$A - 2I = \begin{pmatrix} -9 & 3 \\ -18 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & 0 \end{pmatrix}$$

$$x = \frac{1}{3}y$$

$$\text{let } \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$A + I = \begin{pmatrix} -6 & 3 \\ -18 & 9 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix}$$

$$x = \frac{1}{2}y$$

$$\text{let } \vec{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\text{b) } S = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -7 & 3 \\ -18 & 8 \end{pmatrix}$$

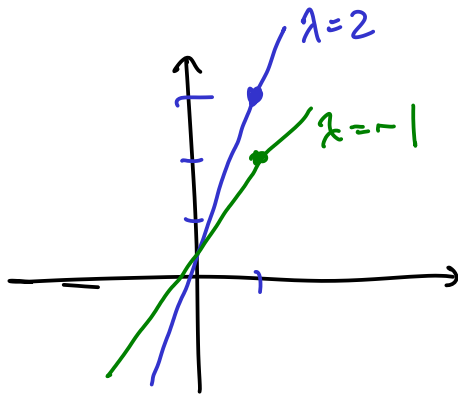
$$AS = \begin{pmatrix} 2 & -1 \\ 6 & -2 \end{pmatrix}$$

$$\begin{matrix} \uparrow & \\ 2\vec{v}_1 & (-1)\vec{v}_2 \end{matrix}$$

$$S^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$$

$$S^{-1}AS = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

c)



dilation by 2 along
the blue line
reflection along the
green line.