Midterm 3 CS 3333 Sp '14

(1) Break the rules of etiquette and seat the men first: $6!$

That gives 7 available spots: $P(7,3) = 7!/4!$

Product rule says $\boxed{6!\,7!/4! = 151200}$

At a round table with men only, division rule gives $\frac{6!}{6} = 5!$

With 6 available spots for women: $5!\,P(6,3) = \boxed{5!\,6!/3! = 14400}$

Now we have 9 spots for the 1st couple, so we get

$14400\,P(9,2) = \boxed{14400 \cdot 9!/7! = 1036800}$

(2) 7 with 4: $n=4$, $k=7-n=3$: $\binom{4+3-1}{3}6^{-4} = 20 \cdot 6^{-4} = \boxed{\frac{5}{324} = .015432\ldots}$

5 with 3: $\binom{3+2-1}{2}6^{-3} = \boxed{6^{-2} = .0277\ldots}$

(3) Let E denote "hair loss" and F spam.

Then $p(F) = .65$, $p(\bar{F}) = .35$, $p(E|F) = .2$, $p(E|\bar{F}) = .01$

By Bayes thm $p(F|E) = \dfrac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})}$

$= \dfrac{.2 \cdot .65}{.2 \cdot .65 + .01 \cdot .35} = .97378\ldots = \boxed{97.4\%}$

(4) $\mu = 10$, $\sigma = 3$ $\quad \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu-2}^{\mu+2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{3\sqrt{2\pi}} \int_8^{12} e^{-\frac{1}{2}\left(\frac{x-10}{3}\right)^2} dx$

Let $z = \frac{x-\mu}{\sigma} = \frac{x-10}{3}$, then $dz = \frac{dx}{3}$, so we get $\frac{1}{\sqrt{2\pi}} \int_{-2/3}^{2/3} e^{-z^2/2} dz =$ ← even function

$2 \cdot \frac{1}{\sqrt{2\pi}} \int_0^{2/3} e^{-z^2/2} dz = \cancel{2} \cdot \frac{1}{\cancel{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\Big|_0^{2/3} = \boxed{\operatorname{erf}\left(\frac{\sqrt{2}}{3}\right) = .4950\ldots}$

erf(0)=0