Midterm 1 CS 3333 Sp '14

(1)

$$\begin{array}{r} 9 \\ 16\overline{)151} \\ 144 \\ \hline 7 \end{array}$$

$\boxed{x97}$ ← hex

$\boxed{\underbrace{10}\underbrace{01\ 0}\underbrace{111}}$ ← binary

$\boxed{2\quad 2\quad 7}$ ← octal

$$x\text{BAD} \qquad 11\cdot\underbrace{16^2}_{256} + 10\cdot 16 + 13$$

$$\begin{array}{r} 256 \\ 11 \\ \hline 256 \\ 256 \\ \hline 2816 \\ 160 \\ \hline 2976 \\ 13 \\ \hline \boxed{2989} \end{array}$$

②

$74 = 2 \cdot 35 + 4$ $\qquad$ $4 = 74 - 2 \cdot 35$

$35 = 8 \cdot 4 + 3$ $\qquad$ $3 = 35 - 8 \cdot 4$

$4 = 3 + \boxed{1}$ $\qquad$ $1 = 4 - 3$

gcd (74, 35)

$$1 = 4 - 3 = 4 - (35 - 8 \cdot 4) = 9 \cdot 4 - 35$$

$$= 9(74 - 2 \cdot 35) - 35 = 9 \cdot 74 - 19 \cdot 35$$

Bézout coeff. $9, \; -19$

③

$2x \equiv 3 \mod 5$

$3 \cdot 2x \equiv 3 \cdot 3 \mod 5$

$x \equiv 4 \mod 5$

Alt: $2x \equiv 3 \mod 5$

$2x \equiv 8 \mod 5$

$x \equiv 4 \mod 5$

$m = 180$

i	$M_i = \frac{m}{m_i}$		M_i^{-1}	a_i	$M_i M_i^{-1} a_i$
1	45	$\equiv 1 \mod 4$	1	2	90
2	36	$\equiv 1 \mod 5$	1	4	144
3	20	$\equiv 2 \mod 9$	5	1	100

$$334 \equiv \boxed{154 \mod 180}$$

④

n	4^n	$n!$
0	1	1
1	4	1
2	16	2
3	64	6
4	256	24
5	1024	120
6	4096	720
7	16,384	5,040
8	65,536	40,320
9	262,144	362,880

☺

<u>Claim:</u> $4^n < n!$ for $n \geq 9$

Basis: $n = 9$ (see above)

Inductive step: Let $n > 9$

$$4^n = 4 \cdot 4^{n-1} \underset{\uparrow}{<} 4 \cdot (n-1)!$$

By induction

$$\underset{\uparrow}{<} n(n-1)! = n!$$

☺

since $n > 4$ $(n > 9)$

⑤

a	b	n
# 2's	# 5's	
0	0	0
0	1	5
1	0	2
1	1	7
0	2	10
2	0	4
3	0	6
4	0	8
2	1	9

Claim : Any amount Except 1, 3 .

Let $P(n)$ = " n is obtainable with \$2's & \$5's"

$P(n)$: $\exists\, a, b \geq 0 \quad n = a2 + b5$

Claim : $P(n)$ is true for $\forall\, n \geq 4$

Basis : $n = 4, 5$ $\quad 4 = 2 \cdot 2$ ☺

$5 = 1 \cdot 5$ ☺

Inductive step : Let $n > 5$

$$n = \underbrace{n - 2}_{\geq 4} + 2$$

By induction $\exists\, a, b \geq 0 \quad n - 2 = a \cdot 2 + b \cdot 5$

$$n = a2 + b \cdot 5 + 2 = (a+1)2 + b5$$ ☺

Alt. Suppose $n \geq 4$.

If n is even $\exists k$ $n = 2k$ ☺

If n is odd $\exists k$ $n = 2k+1$

Since $n \geq 4$, $2k+1 \geq 4$

$2k \geq 3$, so $k \geq 2$

$$n = 2(\underbrace{k-2}_{\geq 0}) + \underbrace{4+1}_{5}$$ ☺