① a)

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$$

b)

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 4 & 0 & 3 & -3 \end{array}\right] \begin{array}{l} \\ r_2 - 2r_1 \\ r_2 - 4r_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & \textcircled{-5} & -3 & -2 \\ 0 & -12 & -5 & -7 \end{array}\right] \begin{array}{l} \\ -\frac{1}{5}r_2 \\ r_3 + 12(\text{new } r_2) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 3/5 & 2/5 \\ 0 & 0 & \textcircled{11/5} & -11/5 \end{array}\right] \begin{array}{l} r_1 - 2(\text{new } r_3) \\ r_2 - \frac{3}{5}(\text{new } r_3) \\ \frac{5}{11}r_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] r_1 - 3r_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\boxed{x_1 = 0, \; x_2 = 1, \; x_3 = -1}$$

c) $\det = -5 \cdot \frac{11}{5} = \boxed{-11}$

(2) a) $A = \begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$ $A - \lambda I = \begin{bmatrix} 5-\lambda & 6 \\ 3 & 2-\lambda \end{bmatrix}$

set $\det(A-\lambda I)=0$, solve for λ

$$(5-\lambda)(2-\lambda) - 3\cdot 6 = 0$$

$$10 - 7\lambda + \lambda^2 - 18 = 0$$

$$\lambda^2 - 7\lambda - 8 = 0$$

$$(\lambda - 8)(\lambda + 1) = 0 \Rightarrow \boxed{\lambda = 8 \text{ or } -1}$$

b) $\lambda = 8$ $A - \lambda I = \begin{bmatrix} -3 & 6 \\ 3 & -6 \end{bmatrix} \xrightarrow{G.J.} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

$x - 2y = 0$

Eigenvector : $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Eigenspace $y = \frac{1}{2}x$

$(= \text{Span}(\vec{v}))$

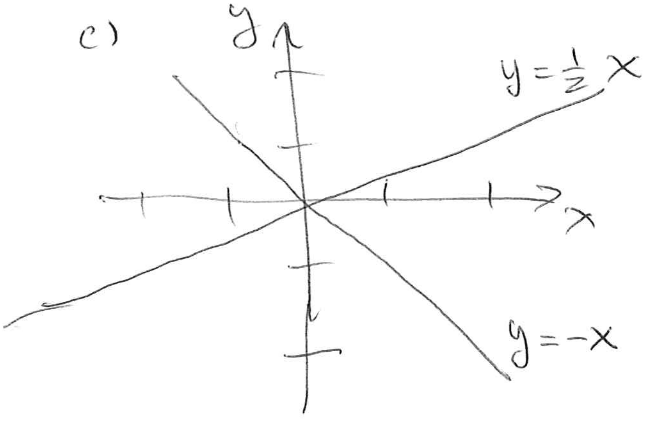
$\lambda = -1$ $A - \lambda I = \begin{bmatrix} 6 & 6 \\ 3 & 3 \end{bmatrix} \xrightarrow{GJ} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$x + y = 0$

Eigenvector : $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Eigenspace $y = -x$

c)



stretch by a factor of 8
along [2, 1],
flip along [1, -1]

(3) a) $P(12,12) = 12! = 479\,001\,600$

b) $12!/12 = 11! = 39\,916\,800$

c) $11! \cdot \underbrace{12 \cdot 11 \cdot 10}_{P(12,3)} = 50\,690\,176\,000$

d) $(n-1)!\, P(n,m) = (n-1)! \dfrac{n!}{(n-m)!}$

(4) $C(3+4-1, 4) = C(6,4) = C(6,2) = \frac{6 \cdot 5}{2} = 15$

$7 = 5+1+1$ 3
+
$7 = 4+2+1$ $3! = 6$
+
$7 = 3+2+2$ 3
+
$7 = 3+3+1$ 3

15 ☺

Total $= 6^3 = 216$

Prob. $= \frac{15}{216} = \boxed{\frac{5}{72} \approx 0.07}$

(5) E = lager, $\bar{E}$ = ale
F = office, $\bar{F}$ = penguin

$P(F) = \frac{1}{3}$, $P(\bar{F}) = \frac{2}{3}$
$p(E|F) = \frac{1}{2}$ $P(\bar{E}|F) = \frac{1}{2}$
$P(E|\bar{F}) = \frac{1}{4}$ $P(\bar{E}|\bar{F}) = \frac{3}{4}$

($P(\bar{F}) = 2P(F)$
$P(\bar{F}) + p(F) = 1 \rightarrow 2P(F) + p(F) = 1$
$3P(F) = 1$
$P(F) = \frac{1}{3}$
$P(\bar{F}) = \frac{2}{3}$)

$$p(F|E) = \frac{\overbrace{p(E|F)\,p(F)}^{\leftarrow 1/6}}{\underbrace{\underbrace{p(E|F)}_{1/2}\,\underbrace{p(F)}_{1/3}}_{1/6} + \underbrace{\underbrace{p(E|\bar{F})}_{\frac{1}{4}}\,\underbrace{p(\bar{F})}_{\frac{2}{3}}}_{1/6}} = \boxed{\frac{1}{2}}$$

(6) Let $z = \frac{x-\mu}{\sigma}$ Bounds: $x = \mu \pm 1$, $x - \mu = \pm 1$, $z = \pm \frac{1}{\sigma}$

$z = \pm \frac{1}{2}$

$$p(-\tfrac{1}{2} \le z \le \tfrac{1}{2}) = 2p(0 \le z \le \tfrac{1}{2}) = 2 \cdot \tfrac{1}{2} \operatorname{erf}\left(\frac{1/2}{\sqrt{2}}\right)$$

↑ By symmetry of $e^{-z^2/2}$ (even)

$$= \boxed{\operatorname{erf}\left(\frac{1}{2\sqrt{2}}\right)}$$