Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Solve the initial value problem $x' = Ax, x(0) = x_0$ using Picard iteration. Try to recognize Picard iterates as approximations to familiar functions.

2. Let $f(x) = \begin{bmatrix} -x_1 \\ -x_2 + x_1^3 \end{bmatrix}$
   (a) Solve the initial value problem $x' = f(x), x(0) = y$ and let $u(t, y)$ denote the solution.
   (b) Compute $\Phi(t, y) = \dfrac{\partial u}{\partial y}$.
   (c) Compute Df.
   (d) Show that Φ satisfies the initial value problem $\Phi' = Df\,\Phi, \Phi(0, y) = I$.

3. Let $J(y)$ denote the maximal time interval of existence for the initial value problem $x' = x^3, x(0) = y$. Let Ω be the subset of the plane defined by $\Omega = \{(t, y) : t \in J(y)\}$. Solve the initial value problem and sketch Ω.

4. Let $f(x) = \begin{bmatrix} -x_1 \\ 2x_2 + x_1^2 \end{bmatrix}$ and $S = \{x \in \mathbf{R}^2 \colon x_2 = -x_1^2/4\}$.
   (a) Find the flow for the system $x' = f(x)$.
   (b) Show that the set S is invariant with respect to the flow.

5. Let $f(x) = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix}$
   (a) Find the equilibrium points of $x' = f(x)$.
   (b) Linearize the system at each equilibrium point.
   (c) Classify each equilibrium point (as a sink, saddle, etc.).

1	2	3	4	5	total (50)