

Name: \_\_\_\_\_

Please show all work and justify your answers.

1. Characterize all finite subgroups of the multiplicative group  $\mathbf{C} \setminus \{0\}$ . Prove your assertion.
2. Find the sizes of conjugacy classes for  $S_4$  and verify the class equation.
3. Let  $p(x) = x^2 + 3x + 1$ ,  $F = \mathbf{Q}[x]/\langle p \rangle$ , and  $u = x + \langle p \rangle \in F$ . Express  $u^3$  and  $(1 + u)^{-1}$  as linear combinations of 1 and  $u$ .
4. In the above problem find the minimal polynomials of  $u^3$  and  $(1 + u)^{-1}$  over  $\mathbf{Q}$ .
5. Find an irreducible polynomial in  $\mathbf{Q}[x]$  whose Galois group over  $\mathbf{Q}$  is isomorphic to the dihedral group  $\Delta_4$ . Prove your assertion.

1	2	3	4	5	total (50)	%

Prelim. course grade: %