Midterm 1 / 2014.3.20 / Algebra II / MAT 5313

Name: ____

Please show all work and justify your answers.

Let K denote a commutative ring with $\mathrm{char}(K) \neq 2$.

1. Use universality of tensor product of K-modules to show $A \otimes B \cong B \otimes A$.
2. Let $d = \gcd(m, n)$. Prove $[x, y] \mapsto xy$ defines a universal $\mathbf{Z}$-bilinear map $\mathbf{Z}_m \times \mathbf{Z}_n \to \mathbf{Z}_d$.
3. Suppose F is free K-module on 2 generators. Prove from first principles that the set $\mathrm{Alt}_2(F)$ of all bilinear alternating maps $F \times F \to K$ is a cyclic K-module.
4. Let M be a K-module. Illustrate with a concrete example that the functor $\_ \otimes M$ is not left exact.
5. Compute the following tensor products over $\mathbf{Z}$. Briefly explain your answers.
   (a) $\mathbf{Z}^2 \otimes \mathbf{Z}^4 \cong$?
   (b) $\mathbf{Z}^2 \otimes \mathbf{Z}_4 \cong$?
   (c) $\mathbf{Z}_2 \otimes \mathbf{Z}_4 \cong$?
   (d) $\mathbf{Z}^3 \otimes \mathbf{Q} \cong$?
   (e) $\mathbf{Z}_3 \otimes \mathbf{Q} \cong$?

1	2	3	4	5	total (50)