Algebra II, MAT 5313
Midterm 2, April 3, 1996
Instructor: D. Gokhman

Name: ______________________________

1	2	3	total (70)

1. (20 pts.) In this problem we will see that the statement

   "THE UNION OF IDEALS IS AN IDEAL"

   is generally false. It is however true if the ideals are assumed to be chained.

   (a) Suppose R is ring and $I_1 \subseteq I_2 \subseteq ...I_k \subseteq I_k$ is a chain of ideals of R. Show that $I = \bigcup_{k=1}^{\infty} I_k$ is an ideal of R.

   (b) Find two ideals I and J of $\mathbf{Z}$ such that $I \bigcup J$ is not an ideal of $\mathbf{Z}$.

2. (30 pts.) Suppose R is an integral domain.

   (a) Show that if P is a prime ideal of R, then $R \setminus P$ is closed under multiplication.

   (b) Show that if R has exactly one maximal ideal M, then $R \setminus M$ is the multiplicative group of units of R.

   (c) Show that R is a field $\Leftrightarrow$ R has no proper nonzero ideals.

3. (20 pts.)

   (a) Find the smallest positive integer x such that

   $$x \equiv 2 \bmod 3 \quad \text{and} \quad x \equiv 3 \bmod 5$$

   (b) For which n is it true that for any solution y of the above system, we have $x \equiv y \bmod n$?