Algebra II, MAT 5313
Midterm 1, March 4, 1996
Instructor: D. Gokhman

Name: ____

1. (20 pts.) Suppose R and S are rings, $f\colon R \to S$ is a ring homomorphism, J is an ideal of R and I is an ideal of S. Prove or disprove:
   (a) $f^{-1}(I)$ is an ideal of R.
   (b) $f(J)$ is an ideal of S.
2. (20 pts.) Suppose R is a commutative ring with 1.
   (a) Prove that the set S of units of R forms a multiplicative group.
   (b) What is the multiplicative group of units of the ring $\mathbf{Z} \times \mathbf{Z}$?
3. (20 pts.) Prove that an ideal of $\mathbf{Z}$ is a prime ideal if and only if it is generated by a prime number.
4. (20 pts.) Suppose R is a commutative ring with 1. Prove that if I is a maximal ideal of R, then I is a prime ideal of R.

1	2	3	4	total (80)