

Name: _____

Please show all work and justify your answers.

1. Integrate $-y \, dy \, dz + x \, dz \, dx + z \, dx \, dy$ over the cone $z^2 = x^2 + y^2$, $0 \leq z \leq 2$.
2. Let $\varphi: A \rightarrow B$ be a morphism in the category of abelian groups. Show that the natural projection $B \rightarrow B/\varphi(A)$ is universal among morphisms from B whose composition with φ is zero.
3. Find the fundamental group of the Mercedes-Benz logo. What can you conclude about the homology groups of the logo?
4. Suppose $A \subseteq X$ is a retract of X , i.e. there exists a continuous map $X \rightarrow A$ whose restriction to A is the identity. Show that the morphisms $H_n(A) \rightarrow H_n(X)$ induced by the inclusion are injective. Show by example that these morphisms need not be surjective. Show by example that injectivity need not hold if A is not a retract of X .
5. Suppose $A \subseteq X$. Show that the inclusion induces isomorphisms of the homology groups of A and X if and only if $H_n(X, A) = 0$ for all n .

1	2	3	4	5	total (50)	%

Prelim. course grade: %