Final exam / 2013.5.8 / MAT 5263.001 / Algebraic topology

Name: ____________________

Please show all work and justify your answers.

1. Describe a universal cover of the torus $S^1 \times S^1$, including the projection map. Explain why this is indeed a universal cover. What are the deck transformations? What does this say about the fundamental group of the torus?

2. For what values of the constants a and b is the 1-form $\omega = (3x^2 + 3y^2 z \sin(xz))\, dx + (ay \cos(xz) + bz)\, dy + (3xy^2 \sin(xz) + 5y)\, dz$ closed? For those values of a and b find all functions whose differential is ω.

3. Find a 1-form on the unit circle S^1 that is closed, but not exact. What conclusions can you draw about the de Rham cohomology of S^1?

4. Represent the torus $S^1 \times S^1$ as a simplicial complex and compute its homology.

5. Let M denote the southern hemisphere of radius 3 centered at the origin. Parametrize M. Evaluate the integral of $y\, dy\, dz - x\, dz\, dx - z\, dx\, dy$ over M.

6. Let $\varphi\colon A \to B$ be a morphism in the category of abelian groups. Show that the inclusion $\ker \varphi \to A$ is universal among morphisms to A whose composition with φ is zero.

7. Find the fundamental group of the pentagram. What can you conclude about the homology of the pentagram?

8. Suppose $A \subseteq X$ is a retract of X, i.e. there exists a continuous map $X \to A$ whose restriction to A is the identity. Show that the morphisms $H_n(A) \to H_n(X)$ induced by the inclusion are injective. Show by example that these morphisms need not be surjective. Show by example that injectivity need not hold if A is not a retract of X.

1	2	3	4	5	6	7	8	total (80)	%