General Topology II, MAT 5253
Final, May 9, 1997
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Name: ____________________ Pseudonym: ____________________

Let I denote the closed unit interval in $\mathbf{R}$ and S^n — the unit sphere in $\mathbf{R}^{n+1}$.
Show all work.

1. (20 pts.) Suppose Y is a connected component of X and $i\colon Y \to X$ is the inclusion map.
   (a) Show that $i_*\colon H_n(Y) \to H_n(X)$ is a 1-1 homomorphism of homology modules.
   (b) Construct a homomorphism $p\colon H_n(X) \to H_n(Y)$ such that $p \circ i = 1_{H_n(Y)}$.
2. (20 pts.) Suppose $X = \mathbf{C} \setminus \{0\}$.
   (a) Let $\sigma\colon I \to X$ be given by $\sigma(s) = e^{2\pi i s}$. Prove that the 1-chain σ is a cycle.
   (b) Consider the following points in X: $z_1 = z_5 = 1$, $z_2 = i$, $z_3 = -1$, $z_4 = -i$. For $k = 1, 2, 3, 4$ let $\sigma_k : I \to X$ be given by $\sigma_k(s) = z_k + s(z_{k+1} - z_k)$ and consider the 1-cycle $\sigma = \sum_{k=1}^{4} \sigma_k$. Find a 2-chain γ such that $\partial\gamma = \sigma$. It is sufficient to specify affine transformations of the plane by their values at three not collinear points.
3. (20 pts.) Suppose X is path connected. Prove that $H_0(X; \mathbf{Z}) \cong \mathbf{Z}$.
4. (20 pts.) Prove that composition of covering maps is a covering map.
5. (20 pts.) True or false — circle your choice. No justification necessary.
   Here E is a covering space of a connected and locally path connected space X.

T F (a) $\pi_1(S^2) \cong \pi_1(S^3)$.
T F (b) $\pi_1(X) \cong H_1(X; \mathbf{Z})$.
T F (c) $\pi_1(X)$ is isomorphic to the group of covering transformations of E.
T F (d) If Y is simply connected, any map $f\colon Y \to X$ can be lifted to E.
T F (e) If X and Y are homotopically equivalent, then $H_n(X) \cong H_n(Y)$.

1	2	3	4	5	total (100)