General Topology I, MAT 5243
Final, December 13, 1996
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Name: ______________________________

Below X and Y denote topological spaces.
Unless otherwise specified, topologies are inherited from Euclidean.
To disprove a statement provide a *specific* counterexample.

1. (16 pts.) Prove or disprove that the topological spaces X and Y are homeomorphic.
   (a) $X = \mathbf{R}^2$, $Y = \mathbf{R}$.
   (b) $X = \{(x, y) \in \mathbf{R}^2 \colon x^2 + y^2 = 1\}$, $Y = \{x \in \mathbf{R} \colon 0 \leq x < 2\pi\}$.
   (c) $X = \mathbf{R}$, $Y = \{x \in \mathbf{R} \colon 0 < x < 1\}$.
   (d) $X = \mathbf{Z}$, $Y = \mathbf{Q}$ with the discrete topology.
2. (12 pts.) Let $X = \mathbf{R}$ with the finite complement topology,
   i.e. $\mathcal{T} = \{U \subseteq \mathbf{R} \colon \mathbf{R} \setminus U \text{ is finite}\} \bigcup \{\emptyset\}$. Prove:
   (a) $\mathcal{T}$ is a topology.
   (b) X is not Hausdorff.
   (c) X is connected.
3. (8 pts.) Suppose A and B are connected and $A \bigcap B \neq \emptyset$. Prove or disprove:
   (a) $A \bigcap B$ is connected.
   (b) $A \bigcup B$ is connected.
4. (4 pts.) Suppose X is connected and $f \colon X \to Y$ is continuous and onto.
   Prove that Y is connected.
5. (4 pts.) Prove that an open connected subset of $\mathbf{R}^n$ is path connected.
6. (8 pts.) Prove:
   (a) X, Y Hausdorff $\Rightarrow X \times Y$ is Hausdorff.
   (b) X, Y path connected $\Rightarrow X \times Y$ is path connected.
7. (12 pts.) True/false questions. Circle your choice. Justification is not necessary.

T F (a) Box topology is finer than the product topology.
T F (b) Box topology is finer than the uniform topology.
T F (c) $\mathbf{R}^\omega$ is metrizable.
T F (d) Products of connected spaces are connected.
T F (e) If $X = A \bigcup B$ and $f \colon X \to Y$ is continuous on A and B, then f is continuous on X.
T F (f) $A \subseteq \mathbf{R}$ is connected $\Leftrightarrow A$ is an interval.

1	2	3	4	5	6	7	total (64)	%