Theory of Functions of a Complex Variable II / MAT 5233
Midterm 2 / April 28, 1999 / Instructor: D. Gokhman

Name: ______________________________

Please show all work.

1. (20 pts.) Let D denote the open unit disc. Show that there is no analytic function $f\colon D \to D$ with $f(0) = 2/3$ and $f'(0) = 5/6$.

2. (20 pts.) Let $G = \{z : |\operatorname{Im} z| < \pi/2\}$ and suppose $f\colon G \to \mathbf{C}$ is analytic and
$$\limsup_{z\to w} |f(z)| \le M \qquad \forall w \in \partial G.$$
Also, suppose $A < \infty$ and $a < 1$ can be found such that
$$|f(z)| < e^{Ae^{a|\operatorname{Re} z|}} \qquad \forall z \in G.$$
Show that $|f(z)| \le M$ for all z in G.

3. (20 pts.) Let $G = \mathbf{C} \setminus \{z \in \mathbf{C}\colon \operatorname{Im} z \in \mathbf{Z}\}$. Sketch G and the sets $K_n = \{z\colon\ |z| \le n, d(z, G^c) \ge 1/n\}$ for $n = 1, 2, 3, 4$.

4. (20 pts.) Suppose G is a domain in $\mathbf{C}$ and (f_n) is a sequence in $\mathcal{C}(G, \mathbf{C})$ which converges to f uniformly on compact subsets of G.
   (a) Show that $f \in \mathcal{C}(G, \mathbf{C})$.
   (b) Show that if $f_n \in \mathcal{H}(G)$ for all n, then $f \in \mathcal{H}(G)$.

1	2	3	4	total (80)	%