Theory of Functions of a Complex Variable II, MAT 5233
Midterm, March 5, 1997
Instructor: D. Gokhman

Name: ____________________

Show all work.

1. (10 pts.) Find all zeros and poles in the Riemann sphere Σ and their order for the rational function
$$f(z) = \frac{z^6 + z^5}{z^4 - 2z^2 + 1}$$

2. (35 pts.) Consider $f\colon \mathbf{C} \to \Sigma$, $f(z) = \tan z$.
   (a) Prove that π is a period of $f(z)$. What is the period group of $f(z)$?
   (b) Find the first two terms of the Laurent expansion of $f(z)$ at $z = 0$. Repeat at $z = \pi/2$.
   (c) Is $f(z)$ meromorphic on $\mathbf{C}$? What are the zeros and poles of $f(z)$ and what is their order?
   (d) Can $f(z)$ be extended to a meromorphic function on Σ? Explain.
   (e) Let $\zeta = e^{iz}$. Find a function $\varphi\colon \mathbf{C} \setminus \{0\} \to \Sigma$ such that $\varphi(\zeta) = f(z)$.
   (f) Expand $\varphi(\zeta)$ in a Laurent series valid for $|\zeta| > 1$.
   (g) Find the corresponding Fourier series expansion of $f(z)$. Where is it valid?

3. (12 pts.) True or false — circle your choice. No justification necessary.

T F (a) $\mathbf{Z}\left[i\sqrt{2}\right] \stackrel{\text{def}}{=} \left\{n + im\sqrt{2}\colon n, m \in \mathbf{Z}\right\}$ is a discrete additive subgroup of $\mathbf{C}$.

T F (b) If Ω is discrete additive subgroup of $\mathbf{C}$, then Ω has no accumulation points.

T F (c) If $f\colon \Sigma \to \Sigma$ is meromorphic, then it is rational.

T F (d) If $f\colon \mathbf{C} \to \Sigma$ is meromorphic and has no poles, then f is constant.

1	2a–d	2e–g	3	total (57)	%