Theory of Functions of a Complex Variable I / MAT 5223.001

Midterm 2 / November 23, 1998 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) For each of the following $f : \mathbf{C} \to \mathbf{C}$ find the set where f is complex differentiable and the set where f is analytic.

$$\text{(a) } f(z) = |z|^2 \quad \text{(b) } f(z) = (\overline{z} + i)^2$$

2. (15 pts.) Let $f(z) = 1/(2i - z)$.
   (a) Find the set $\{z \in \mathbf{C}\colon f \text{ is complex differentiable at } z\}$.
   (b) Expand $f(z)$ in a power series at the origin.
   (c) Find the radius of convergence of this power series.

3. (20 pts.) Evaluate the following integrals.
   (You may use the Cauchy Integral Formula, where applicable.)
   (a) $\displaystyle\int_\gamma \frac{\cos(z)}{z^3}\, dz$, where $\gamma = \left\{i + 2e^{i\theta}\colon -\pi \le \theta \le \pi\right\}$,
   (b) $\displaystyle\int_\gamma \frac{dz}{z^3 + z}$, where $\gamma = \left\{2e^{i\theta}\colon -\pi \le \theta \le \pi\right\}$,
   (c) $\displaystyle\int_\gamma \frac{dz}{2z + i}$, where γ is the unit circle traversed counterclockwise,
   (d) $\displaystyle\int_\gamma \operatorname{Re} z\, dz$, where γ is the segment from -1 to i.

4. (10 pts.) Suppose $f\colon \mathbf{C} \to \mathbf{C}$ is analytic and $\exists a \in \mathbf{R} \quad \forall z \in \mathbf{C} \quad |f(z)| \le a\,|z|$.
   Prove that f is linear.

1	2	3	4	total (55)	%

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