Theory of Functions of a Complex Variable I / MAT 5223.001 Midterm 1 / October 14, 1998 / Instructor: D. Gokhman

Name:

- 1. (5 pts.) Find and sketch all $z \in \mathbf{C}$ such that $z^3 = -8$.
- 2. (10 pts.) Suppose Re $(z^n) \ge 0$ for all $n \in \mathbf{N}$. Prove that $z \in \mathbf{R}$.
- 3. (10 pts.) Prove that $\{z \in \mathbb{C} : \operatorname{Re} z > 0\}$ is open in \mathbb{C} . Sketch this set.
- 4. (10 pts.) Sketch each of the following subsets of **C** and answer these questions: is the set (i) open in **C**? (ii) closed in **C**? (iii) connected?

(a)
$$\{x + iy: x^2 \neq y^2\}$$
 (b) $\{z: |z| = |z - 2i|\}$

5. (20 pts.) Suppose $K \subseteq \mathbf{C}$ is compact and $w \in \mathbf{C} \setminus K$. Prove that d(w, K) > 0.

1	2	3	4	5	total (55)	%