

Name: _____

1. (5 pts.) Find and sketch all $z \in \mathbf{C}$ such that $z^3 = -8$.
2. (10 pts.) Suppose $\operatorname{Re}(z^n) \geq 0$ for all $n \in \mathbf{N}$. Prove that $z \in \mathbf{R}$.
3. (10 pts.) Prove that $\{z \in \mathbf{C}: \operatorname{Re} z > 0\}$ is open in \mathbf{C} . Sketch this set.
4. (10 pts.) Sketch each of the following subsets of \mathbf{C} and answer these questions: is the set (i) open in \mathbf{C} ? (ii) closed in \mathbf{C} ? (iii) connected?

$$(a) \{x + iy: x^2 \neq y^2\} \quad (b) \{z: |z| = |z - 2i|\}$$

5. (20 pts.) Suppose $K \subseteq \mathbf{C}$ is compact and $w \in \mathbf{C} \setminus K$. Prove that $d(w, K) > 0$.

1	2	3	4	5	total (55)	%