Theory of Functions of a Complex Va riable I/ MAT5223.001
Midterm 1 / October 14, 1998 / Instructor: D. Gokhman

Name: $\qquad$

1. (5 pts.) Find and sketch all $z \in \mathbf{C}$ such that $z^{3}=-8$.
2. (10 pts.) Suppose $\operatorname{Re}\left(z^{n}\right) \geq 0$ for all $n \in \mathbf{N}$. Prove that $z \in \mathbf{R}$.
3. (10 pts.) Prove that $\{z \in \mathbf{C}: \operatorname{Re} z>0\}$ is open in C. Sketch this set.
4. (10 pts.) Sketch each of the following subsets of $\mathbf{C}$ and answer these questions: is the set (i) open in C? (ii) closed in C? (iii) connected?
(a) $\left\{x+i y: x^{2} \neq y^{2}\right\}$
(b) $\{z:|z|=|z-2 i|\}$
5. (20 pts.) Suppose $K \subseteq \mathbf{C}$ is compact and $w \in \mathbf{C} \backslash K$. Prove that $d(w, K)>0$.

| 1 | 2 | 3 | 4 | 5 | total (55) | $\%$ |
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