Theory of Functions of a Complex Variable I, MAT 5223

Final, December 9, 1996

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Name: ______________________________

1. (28 pts.) Integrate counterclockwise over the unit circle. Show your work.

   Hints: In (c–e) you might want to use Cauchy's Integral Formula and in (f–g) it may help to find the first few terms of the Laurent series expansion of the integrand.

   (a) $\int z^{-n}\,dz, \quad n \in \mathbf{Z}$ (b) $\int z\log(z)\,dz$

   (c) $\int \frac{1}{(2z^2 - z)}\,dz$ (d) $\int \frac{z}{(2z+1)^2}\,dz$ (e) $\int \frac{1}{(z^2+4)^3}\,dz$

   (f) $\int z^{-7}\exp\left(z^3\right)\,dz$ (g) $\int \frac{z}{\sin(2z^2)}\,dz$

2. (8 pts.) Expand $\frac{1}{1-z}$ in a Taylor series at 0. Show that this series converges uniformly on any compact subset of the open unit disc.

3. (8 pts.) Suppose f is entire, $f(0) = 0$, and $f \not\equiv 0$. Show that there exists an open neighborhood of 0 containing no other zeros of f.

4. (12 pts.) True/false questions. Circle your choice. Justification is not required.

T F (a) An entire function is infinitely differentiable.

T F (b) There are no bounded nonconstant entire functions.

T F (c) Every complex polynomial has a zero.

T F (d) Uniform limit of holomorphic functions is holomorphic.

T F (e) If f is holomorphic on a domain Ω, then the integral of f around any closed rectifiable curve in Ω is 0.

T F (f) If the integral of f around any closed rectifiable curve in a domain Ω is 0, then f is holomorphic on Ω.

1a–b	1c–e	1f–g	2	3	4	total (56)	%