University of Texas at San Antonio

Complex Variable I, MAT 5223
Exam №2, 11/23/92
Instructor: D. Gokhman

Name: ____________________

1. (20 pts.)

   (a) Classify all functions $f : \mathbf{C} \to \mathbf{C}$ such that f and $\overline{f}$ are analytic.

   (b) Show that if $f : \mathbf{C} \to \mathbf{C}$ is analytic and $|f|$ is constant, then f is constant.

2. (54 pts.) For the following functions $f(z)$ and curves γ

   (i) Find a parametrization for γ.

   (ii) Calculate $V(\gamma)$.

   (iii) Calculate $\int_\gamma f(z)\,dz$.

   (a) $f(z) = \frac{z+2}{z}$ and γ is given by $\{z\colon\ |z| = 2, \operatorname{Re} z \geq 0\}$.

   (b) $f(z) = \frac{z+2}{z}$ and γ is given by $\{z\colon\ |z| = 2, \operatorname{Re} z \leq 0\}$.

   (c) $f(z) = z - 1$ and γ is given by the straight line segment from 0 to 2.

3. (26 pts.) In problem 2

   (i) Check your answer in part (c) by finding an antiderivative of $f(z)$ and applying the Fundamental Theorem of Calculus.

   (ii) Explain why the Fundamental Theorem of Calculus does not apply to parts (a) and (b).

   (iii) Show how the Cauchy integral formula can be applied to obtain the difference between parts (a) and (b).