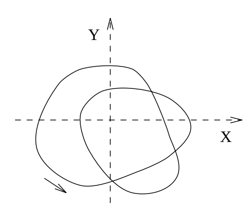
Complex Variable I, MAT 5223. Final Exam. 12/17/93
Instructor: D. Gokhman

Show all pertinent work, answers alone are not sufficient. Box the answers.
All questions carry equal weight.

Name: ______________________________

1. Suppose $f(z)$ is entire. Prove that $\overline{f\left(\overline{z}\right)}$ is entire.
2. Find an analytic function $f(z) = f(x+iy)$ such that $u(x,y) = \operatorname{Re} f(z) = xy$. Express f as a function of z.
3. Consider the map $f(z) = 1/z$. Determine the image of the line $\operatorname{Im} z = 1$. Sketch. Explain.
4. Consider the power series
$$\sum_{n=1}^{\infty} z^n.$$
   (a) Find the radius of convergence.
   (b) Prove that convergence is uniform in any disk centered at the origin with radius smaller than the radius of convergence.
5. (a) Find a parametrization for the straight line segment from 0 to $5 - 2i$.
   (b) Integrate $\operatorname{Im} z$ along this segment.
6. Calculate the following curve integrals:
$$(a)\ \int_\gamma \frac{dz}{z^2+4}, \text{ where } \gamma \text{ is circle of radius 5 centered at 0.}$$
$$(b)\ \int_\gamma \frac{\cos z\, dz}{z^3}, \text{ where } \gamma \text{ is:}$$



1	2	3	4	5	6	total (120)