University of Texas at San Antonio

Complex Variable I, MAT 5223
Final, 12/07/92
Instructor: D. Gokhman

Name: ______________________________

1. (30 pts.) Suppose z_1, z_2, z_3 belong to the unit circle and $z_1 + z_2 + z_3 = 0$. Prove that the triangle with vertices z_1, z_2, z_3 is equilateral.

2. (20 pts.) Suppose $f(z)$ is entire. Prove that so is $\overline{f(\overline{z})}$.

3. (30 pts.) Consider the map $f(z) = 1/z$. Determine (with proof) the images of the lines $\operatorname{Re} z = 0$ and $\operatorname{Re} z = 1$. Sketch.

4. (40 pts.) Consider the power series

$$\sum_{n=1}^{\infty} \frac{z^n}{n^2}.$$

   (a) Find the radius of convergence.

   (b) Prove that convergence is uniform within the radius of convergence.

5. (40 pts.)

   (a) Find a parametrization for the straight line segment from 0 to $2 + i$.

   (b) Integrate $\operatorname{Im} z$ along this segment.

6. (40 pts.) Calculate the following curve integrals:

$$(a) \int_\gamma \frac{dz}{(z^2-1)^3}, \text{ where } \gamma \text{ is circle of radius 5 centered at 0.}$$

$$(b) \int_\gamma \frac{\sin z dz}{z^4}, \text{ where } \gamma \text{ is:}$$