Midterm 2 / 2013.11.26 / MAT 5173.001 / Algebra I

Name: ____

Please show all work and justify your answers.

1. Suppose R is a commutative ring with 1 and let $U(R)$ denote the set of all units in R.
   (a) Prove $U(R)$ is a multiplicative group.
   (b) Prove R is a local ring (has exactly one maximal ideal) if and only if $R \setminus U(R)$ is an ideal of R.
   Hint: By Zorn's Lemma any proper ideal of R is contained in a maximal one.
2. Suppose G is an additive group. Fix an integer $m > 1$ and let $H = \{g \in G : mg = 0\}$. Prove the following:
   (a) $H < G$
   (b) $\mathrm{Hom}(\mathbf{Z}, G) \cong G$
   (c) $\mathrm{Hom}(\mathbf{Z}_m, G) \cong H$
3. In the above problem with $G = \mathbf{Z}_n$ prove H is cyclic. What is its size?
4. If R is a ring with 1 and X is a set, find the R-module dual to the free R-module on X.
5. Prove that coproduct of a nonempty family of free R-modules is free.

1	2	3	4	5	total (50)	%

Prelim. course grade: %