Midterm 2 / 2003.11.26 / Algebra I / MAT 5173

Name: ____

Please show all work and justify your answers.

1. Let $\mathcal{L}$ be the lattice of all submodules of an R-module M and let $B \in \mathcal{L}$. In each case, determine whether $\varphi\colon \mathcal{L} \to \mathcal{L}$ must be a lattice morphism. If so, prove it. If not, construct an explicit counterexample, where φ violates a lattice morphism axiom.
   (a) $\varphi(A) = A \cap B$.
   (b) $\varphi(A) = A + B$.
2. Suppose $f\colon A \to B$ is an R-module morphism. State and prove the universal property of the projection $p\colon B \to \operatorname{coker} f$.
3. Suppose K is a principal ideal domain. Prove that all K-submodules of K are free.
4. Prove that the dual of an R-module epimorphism is a monomorphism.
5. Prove that the dual R-module $(R^2)^*$ is free. Hint: coordinate projections.

1	2	3	4	5	total (50)	%

Prelim. course grade: %

THE UNIVERSITY OF TEXAS AT SAN ANTONIO