Name: _____

Please show all work and justify your answers.

1. How many group endomorphisms does $\mathbf{Z}_4$ have? Exhibit all group automorphisms of $\mathbf{Z}_4$. What famous group is $\mathrm{Aut}(\mathbf{Z}_4)$ isomorphic to? Explain.

2. Consider the additive group $\mathbf{R}^2$. Let H be the subgroup generated by $(1,2)$. Describe all cosets of H. Sketch H and a few cosets.

3. Suppose G and H are groups and $f\colon G \to H$ is a group morphism.

   (a) Prove that the kernel of f is a normal subgroup of G.

   (b) Provide a *concrete* example to illustrate (with proof) that the image of f need not be normal in H.

4. How many ring morphisms $\mathbf{Z}_2 \to \mathbf{Z}$ are there? Group morphisms? Explain.

5. How many ring automorphisms of $\mathbf{Z}_2[x]$ are there? Explain.

6. Let J be the ideal of $\mathbf{Q}[x]$ generated by $x + 1$. Prove that $\mathbf{Q}[x]/J \cong \mathbf{Q}$.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %