Algebra I, MAT 5173
Midterm, October 18, 1995
Instructor: D. Gokhman

Name: _____

1. (10 pts.) Give an example of a semigroup which is not a monoid and an example of a monoid which is not a group.

2. (30 pts.) Sketch the lattice of all subgroups for each of the following groups:

   (a) $(\mathbf{Z}_4, +)$

   (b) $(\mathbf{Z}_2 \times \mathbf{Z}_2, +)$

   (c) S_3 (it may help to label the elements of this group)

3. (20 pts.) Prove that $(\mathbf{Z}_{80}, +)$ has exactly one subgroup of order 16. What is this subgroup?

4. (20 pts.) Suppose G and H are groups and $f: G \to H$ is a group homomorphism. Prove that the kernel of f is a subgroup of G and $f(G) < H$.

5. (40 pts.) Let $f: (\mathbf{R}, +) \to Gl_2(\mathbf{R})$ be given by

$$f(t) = \begin{pmatrix} \cos 2\pi t & -\sin 2\pi t \\ \sin 2\pi t & \cos 2\pi t \end{pmatrix}$$

   (a) Prove that for each t the matrix $f(t)$ is indeed invertible.

   (b) Prove that f is a group homomorphism.

   (c) What subgroup of $\mathbf{R}$ is the kernel of f?

   (d) Prove or disprove that f is onto.

1	2	3	4	5	total (120)