Algebra I, MAT 5173
Final, December 13, 1995
Instructor: D. Gokhman

Name: ____________________ Pseudonym: ____________________

1. (20 pts.) Suppose the order p of a group G is prime. Prove that $G \cong \mathbf{Z}_p$.
2. (24 pts.) Let H be the subgroup of S_4 generated by $(1, 2, 3)$. Compute the left cosets of H in S_4.
3. (20 pts.) Let $N = \{\sigma \in S_5 \colon \sigma(2) = 2\}$. Prove that $N < S_5$ and $N \not\triangleleft S_5$.
4. (20 pts.) Prove that $A_n \triangleleft S_n$
5. (30 pts.) Suppose $f : G \to H$ is a group homomorphism, $K = \ker f$ and $N < G$. Prove that
   (a) $f^{-1}(f(N)) = KN$
   (b) if H is abelian and $K < N$, then $N \triangleleft G$.
6. (20 pts.) What is the free group on $\{x\}$? Justify your answer.
7. (30 pts.) True or false (circle your choice, no justification necessary):

T F (a) Suppose $H \triangleleft G$. If H and G/H are finitely generated, then G is finitely generated.

T F (b) If $H, K < G$, then $HK < G$

T F (c) Every permutation is a product of disjoint cycles.

T F (d) Every cycle is a product of 2-cycles.

T F (e) Suppose G and H are abelian groups. There are more morphisms from G to H in the category of groups than in the category of abelian groups.

1	2	3	4	5	6	7	total (164)