Topology — MAT 4273, Fall 1997

Midterm, 1997/10/15 — Instructor: D. Gokhman

Name: ______________________________

1. (20 pts.) Suppose $f\colon X\to Y$ is a function, $A\subseteq X$, and $B\subseteq Y$.
   (a) Prove that $X\setminus f^{-1}(B)=f^{-1}(Y\setminus B)$.
   (b) Disprove by counterexample that $Y\setminus f(A)=f(X\setminus A)$.

2. (20 pts.) Let $A=\{(x,y)\in\mathbf{R}^2\colon x>1\}$ Define $f\colon\mathbf{R}^2\to\mathbf{R}^2$ by
$$f(x,y)=\begin{cases}(x,y) & \text{if } (x,y)\in A\\ (0,0) & \text{if } (x,y)\notin A\end{cases}$$
   (a) Sketch A. Prove that A is open in $\mathbf{R}^2$.
   (b) Show that f is not continuous and illustrate with a sketch.

3. (25 pts.) In each case give an example or state that there can be no such example.
   (a) A collection of open subsets of $\mathbf{R}^2$ whose union is not open.
   (b) A collection of open subsets of $\mathbf{R}^2$ whose intersection is not open.
   (c) An open cover for a set A without a finite subcover, where
      (i) $A=\{u\in\mathbf{R}^2\colon 0<|u|\le 1\}$
      (ii) $A=\{u\in\mathbf{R}^2\colon |u|\le 1\}$
      (iii) $A=\{u\in\mathbf{R}^2\colon |u|<1\}$

4. (20 pts.) True/false — circle your choice. Justification is not required. Throughout this problem f is a <u>continuous</u> function.

T F (a) $f(A)\subseteq B\Leftrightarrow A\subseteq f^{-1}(B)$.

T F (b) If X is compact, then $f(X)$ is compact.

T F (c) If $f\colon\mathbf{R}\to\mathbf{R}$, then there exists $x\in\mathbf{R}$ such that $f(x)=0$.

T F (d) A subset A of $\mathbf{R}^2$ is compact $\Leftrightarrow$ A is closed and bounded.

T F (e) A subset A of $\mathbf{R}^2$ is closed $\Leftrightarrow$ A is not open.

1	2	3	4	total (85)	%