Name: ______________________ Pseudonym: ______________________

Please show all work. (Notation: $S^n = \{u \in \mathbf{R}^{n+1}\colon\ |u| = 1\}$, $D^n = \{u \in \mathbf{R}^n\colon\ |u| \le 1\}$)

1. (15 pts.) Let $A = \{(x, y) \in \mathbf{R}^2\colon x - y \ge 1\}$. Define $f\colon \mathbf{R}^2 \to \mathbf{R}^2$ by
$$f(x,y) = \begin{cases} (x,y) & \text{if } (x,y) \in A \\ (0,0) & \text{if } (x,y) \notin A \end{cases}$$
   (a) Sketch A.
   (b) Describe or sketch the set of points of discontinuity of f.
   (c) Illustrate the discontinuity of f with a sketch.

2. (30 pts.) In each case construct an example or state that there can be no such example.
   (a) A collection of closed subsets of the plane whose union is not closed.
   (b) $A = f^{-1}(B)$, where f is continuous, A is not connected, and B is connected.
   (c) $A = f^{-1}(B)$, where f is continuous, A is not compact, and B is compact.
   (d) A continuous $f\colon D^2 \to S^1$ such that f restricted to S^1 is the identity function.
   (e) An open cover for a set A without a finite subcover, where
   (i) $A = \{u \in \mathbf{R}^2\colon 1 < |u| \le 2\}$ (ii) $A = \{u \in \mathbf{R}^2\colon 1 \le |u|\}$

3. (10 pts.) Are the following spaces connected? If not, construct a separation.
   (a) $\{(x, y) \in \mathbf{R}^2\colon y = mx,\ \text{where } m \in \mathbf{Q}\}$ (b) $\{(x, y) \in \mathbf{R}^2\colon x + y \in \mathbf{Q}\}$

4. (15 pts.) If X and Y are homeomorphic, construct a homeomorphism $f\colon X \to Y$. If X and Y are not homeomorphic, why not?
   (a) $X = \mathbf{R}$, $Y = (0, 1)$. (b) $X = \mathbf{R}$, $Y = S^1$. (c) $X = S^1$, $Y = S^2$.

5. (15 pts.) For the following transformations of the plane f compute the winding number of the image of the unit circle relative to the origin.
   (a) $f(x, y) = (-x, y)$ (b) $f(x, y) = (-x, -y)$ (c) $f(x, y) = (x - 2, y)$

6. (5 pts.) Construct a homotopy in the plane from the unit circle to the origin.

7. (30 pts.) True/false — circle your choice. Justification is not required.

T F (a) If a map f is continuous, 1-1, onto, then f is a homeomorphism.
T F (b) $f(A) \subseteq B \Leftrightarrow A \subseteq f^{-1}(B)$.
T F (c) A subset A of $\mathbf{R}^2$ is closed $\Leftrightarrow$ $\mathbf{R}^2 \setminus A$ is open.
T F (d) A subset A of $\mathbf{R}^2$ is not compact $\Leftrightarrow$ A is closed but not bounded.
T F (e) If $f\colon S^1 \to S^1$ is continuous, then there exists p with $f(p) = p$.
T F (f) Any two curves in the plane are homotopic.

Have a great break! *– D*

1	2	3	4	5	6	7	total (120)