Name: _____

Please show all work and justify your statements.

1. How many solutions does the congruence $x^{24} + 22x^2 \equiv 0 \bmod 23$ have? Explain.

2. Suppose a belongs to the exponent k modulo p. Find an explicit formula for the multiplicative inverse of a modulo p.

3. Find all real x such that $[2x] = 2$. Do the same for $[2x] = 2[x]$.

4. Prove that $\displaystyle\sum_{d|n} \mu(d)\varphi(d^2) = \prod_{p|n}(1 + p - p^2)$.

5. Suppose u_n is a sequence where each entry, other than u_0 and u_1, is the average of the two preceding entries. Find a formula for u_n in terms of u_0 and u_1. What is the limit of u_n as $n \to \infty$?

6. Find all integer solutions of $6x - 10y = 22$.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %